Marine Systems & Robotics
Unit 02 – AUV Modelling

impact Erasmus+

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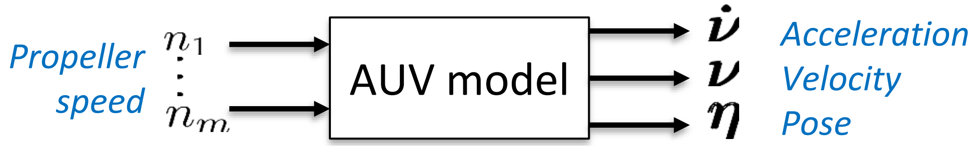
Introduction

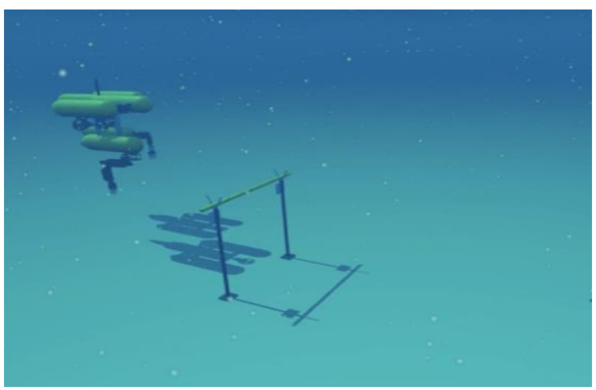
What is AUV Modelling?

- Mathematical Equation of Motion computing the robot position, velocity and acceleration given a control input (thruster velocity f.i.)



What is it needed for?

- **Simulation:** It allows to simulate the behaviour of the robot in a computer



- **Control:** It allows to use control methods that account for the robot dynamics

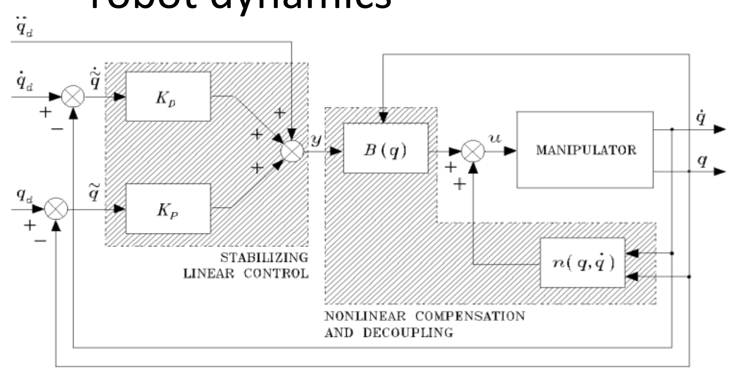


Fig. 8.22. Block scheme of joint space inverse dynamics control

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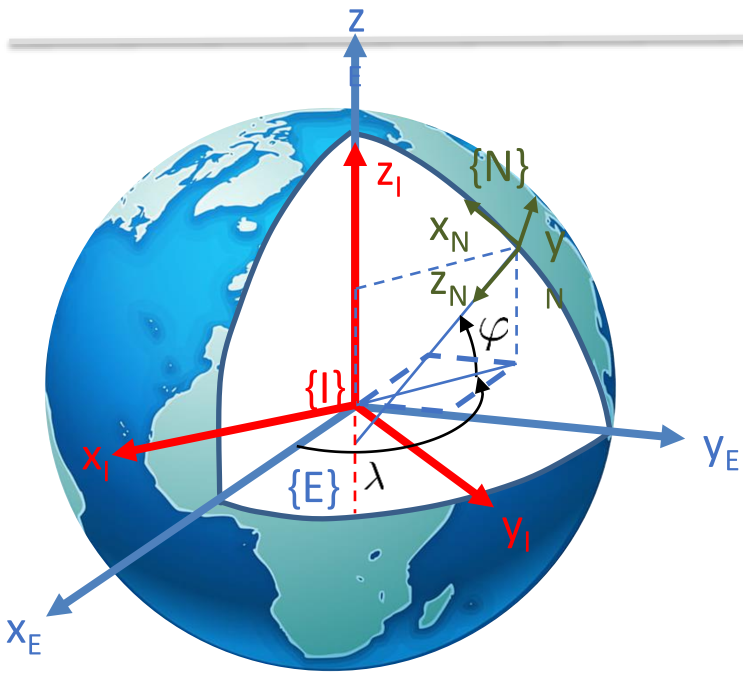
Reference frames



{I} Origin at the centre of the earth
Inertial
Non-rotating wrt sky fixed stars

{E} Origin at the centre of the earth
Rotates with the earth
$\boldsymbol{\omega}^{i}_{ie}=[0\ 0\ \Omega]^T$

{N} Origin at P on the earth surface
Plane XY tg to earth surface
P is a mobile point
Axis pointing North-East-Down
$\boldsymbol{\omega}^{n}_{en}\neq\mathbf{0}$

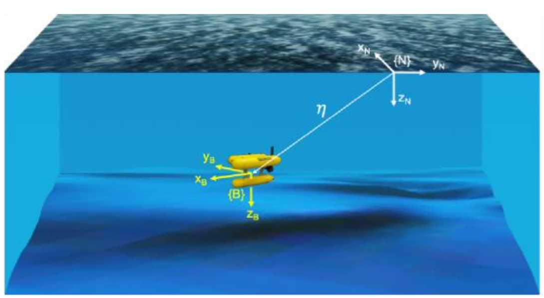
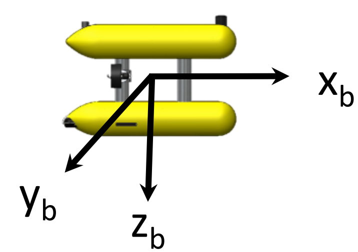
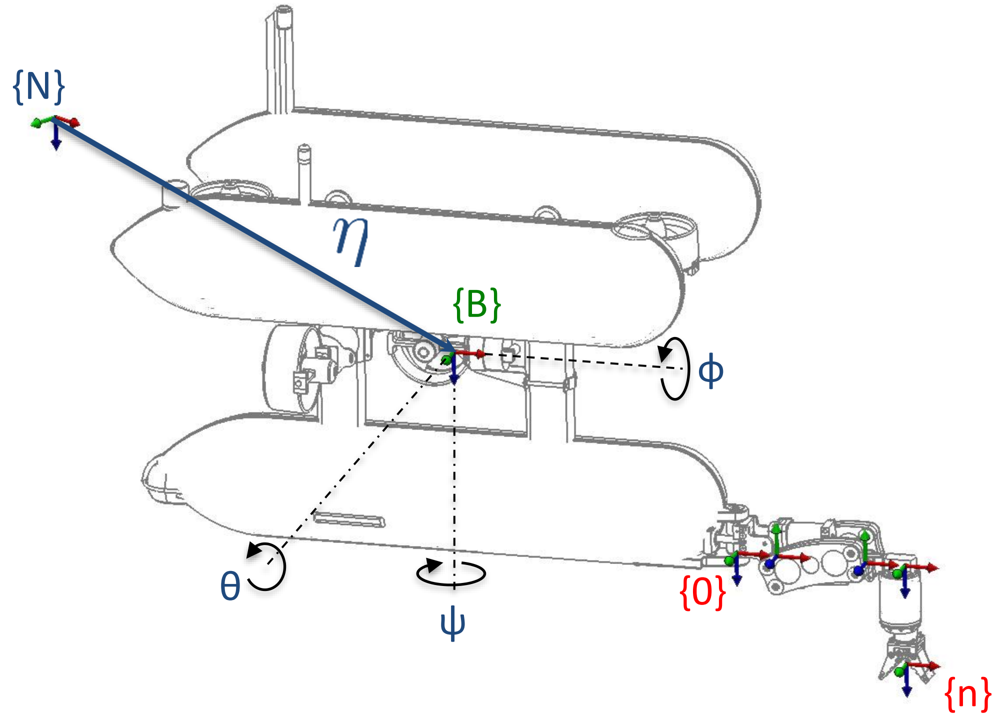


Fig. 1.2. Reference frames: NED ($N-frame$) and body-fixed ($B-frame$))

{B} Vehicle Body fixed frame



AUV Kinematics of Position



- $\eta_1 = [x\ y\ z]^T \in \mathbb{R}^3$: vehicle position in the N-frame.
- $\eta_2 = [\phi\ \theta\ \psi]^T \in \mathbb{R}^3$: vehicle RPY attitude.
- $\eta = [\eta_1^T\ \eta_2^T]^T = [x\ y\ z\ \phi\ \theta\ \psi]^T$: vehicle pose in N-frame.

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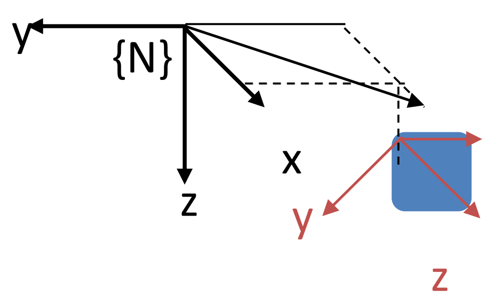
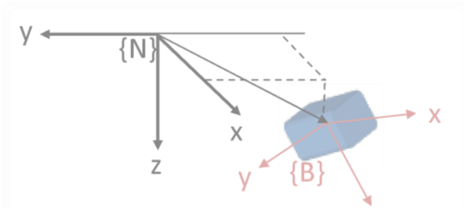
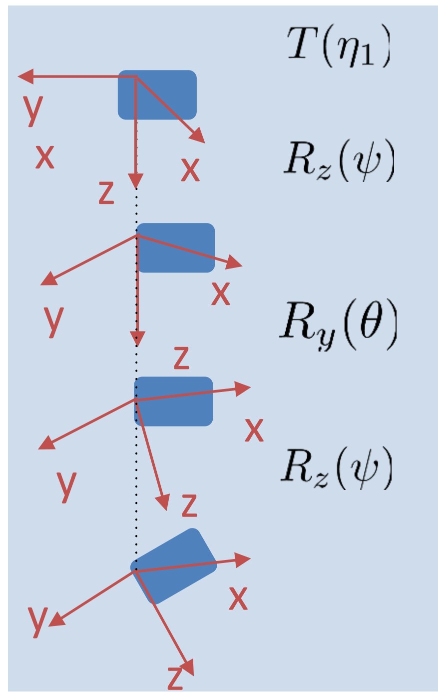
AUV Kinematics: 6DOF Pose

$$
{}^{N}K_B(\eta) = \overbrace{\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{T(\eta_1)}
\overbrace{\begin{bmatrix} c_\psi & -s_\psi & 0 & 0 \\ s_\psi & c_\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_z(\psi)}
\overbrace{\begin{bmatrix} c_\theta & 0 & s_\theta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\theta & 0 & c_\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_y(\theta)}
\overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\phi & -s_\phi & 0 \\ 0 & s_\phi & c_\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_x(\phi)}
= \overbrace{\begin{bmatrix} c_\psi c_\theta & c_\psi s_\phi s_\theta - c_\phi s_\psi & s_\phi s_\psi + c_\phi c_\psi s_\theta & x \\ c_\theta s_\psi & c_\phi c_\psi + s_\phi s_\psi s_\theta & c_\phi s_\psi s_\theta - c_\psi s_\phi & y \\ -s_\theta & c_\theta s_\phi & c_\phi c_\theta & z \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{{}^{N}K_B(\eta)}
$$







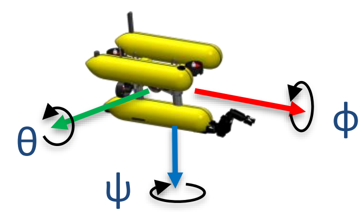
How to represent the AUV pose?

Homogeneous Matrix

$$
{}^{N}K_B(\eta) = \begin{bmatrix} {}^{N}R_B(\eta_2) & \eta_1 \\ 0_{1\times 3} & 1 \end{bmatrix}
$$

Pose Vector

$$
\eta = [x \; y \; z \; \phi \; \theta \; \psi]^T
$$



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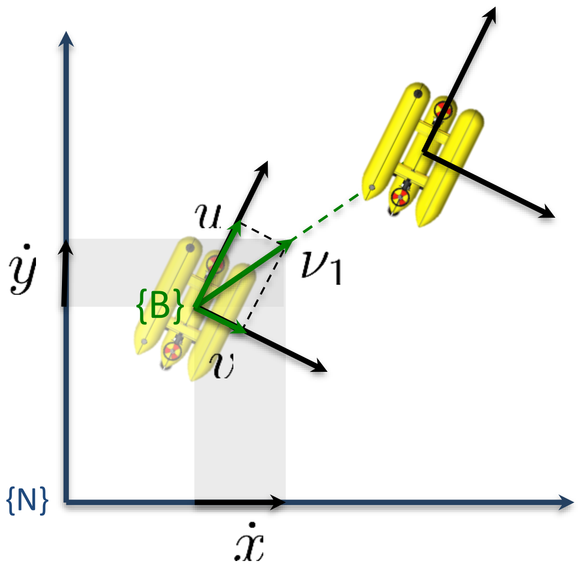
AUV Kinematics of Velocity



$$\dot{\eta_1} = {}^N R_B \cdot \nu_1$$
$$\dot{\eta_2} = J_{\nu_2}(\eta_2) \cdot \nu_2$$
$$\dot{\eta} = J_v(\eta)\nu$$

Where

$$J_v(\eta) = \begin{bmatrix} {}^N R_B(\eta_2) & 0_{3\times3} \\ 0_{3\times3} & J_{\nu_2}(\eta_2) \end{bmatrix}$$

- $\nu_1 = [u\ v\ w]^T \in \mathbb{R}^3$: linear velocity in the B-frame.
- $\nu_2 = [p\ q\ r]^T \in \mathbb{R}^3$: angular velocity in the B-frame.
- $\nu = [\nu_1^T\ \nu_2^T]^T = [u\ v\ w\ p\ q\ r]^T$: velocitiy in B-frame.
- $\dot{\eta} = [\dot{\eta_1}^T\ \dot{\eta_2}^T]^T = [\dot{x}\ \dot{y}\ \dot{z}\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}]^T$: vehicle pose derivative in N-frame.

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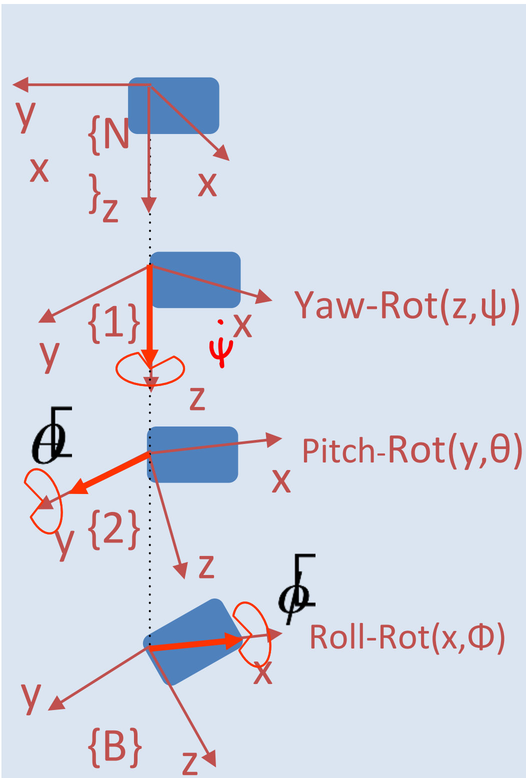
AUV Kinematics of Velocity



$$\dot{\eta_2} = J_{\nu_2}(\eta_2) \cdot \nu_2$$

How can we compute $J_{\nu_2}(\eta_2)$?

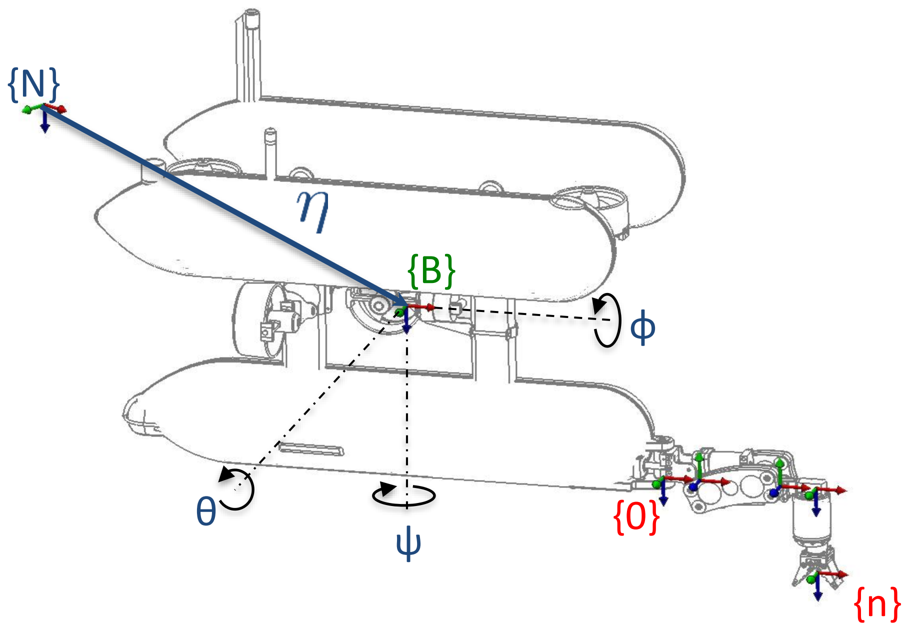
$$\nu_2 = {}^{B}\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \underbrace{R^T_{\phi,x}}_{{}^{B}R_2} {}^{2}\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \underbrace{(R_{\theta,y} R_{\phi,x})^T}_{{}^{B}R_1} {}^{1}\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} =$$

$$\begin{bmatrix} \dot{\phi} - s_\theta \dot{\psi} \\ c_\phi \dot{\theta} + s_\phi c_\theta \dot{\psi} \\ -s_\phi \dot{\theta} + c_\phi c_\theta \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\theta & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} =$$

$$= J_{\nu_2}(\eta_2)^{-1} \dot{\eta_2}$$

$$J_{\nu_2}(\eta_2) = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix}$$

AUV Kinematics Summary



Vehicle

Position

$$^{N}K_B(\eta) = \begin{bmatrix} ^{N}R_B(\eta_2) & \eta_1 \\ 0_{1\times 3} & 1 \end{bmatrix}$$

$$^{N}R_B(\eta_2) = R_{z,\psi} \cdot R_{y,\theta} \cdot R_{x,\phi}$$

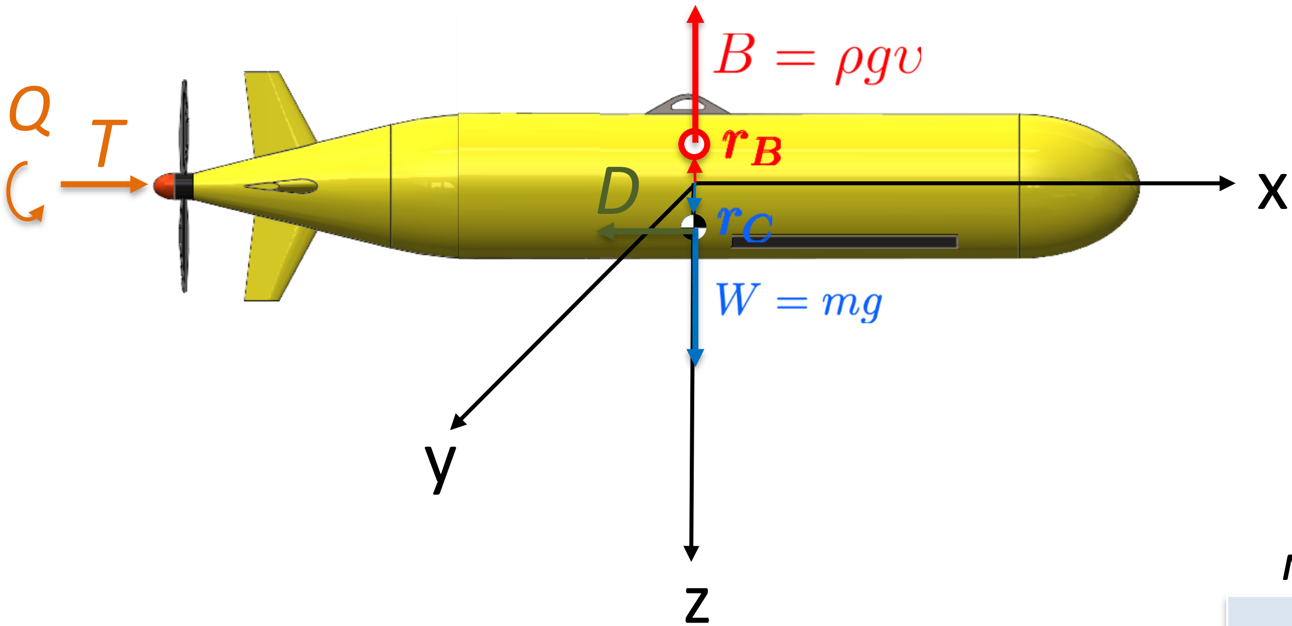
Velocity

$$\dot{\eta} = J_v(\eta)\nu$$

$$J_v(\eta) = \begin{bmatrix} ^{N}R_B(\eta_2) & 0_{3\times 3} \\ 0_{3\times 3} & J_{\nu_2}(\eta_2) \end{bmatrix}$$

- $\eta = [\eta_1^T \; \eta_2^T]^T = [x \; y \; z \; \phi \; \theta \; \psi]^T$: vehicle pose in N-frame.
- $\nu = [\nu_1^T \; \nu_2^T]^T = [u \; v \; w \; p \; q \; r]^T$: velocitiy in B-frame.
- $\dot{\eta} = [\dot{\eta_1}^T \; \dot{\eta_2}^T]^T = [\dot{x} \; \dot{y} \; \dot{z} \; \dot{\phi} \; \dot{\theta} \; \dot{\psi}]^T$: vehicle pose derivative in N-frame.

Rigid Body Dynamics



- r_C Gravity center
- r_B Buoyancy center
- W Weighty
- B Buoyancy
- T Propeller Thrust
- Q Propeller Torque
- Damping (Skin Friction)

Forces

- Propulsion
- Control Surfaces
- Restoring Forces (Gravity & Buoyancy)
- Hydrodynamic Forces
  - Added Mass
  - Friction
- Environmental (Currents)

Newton Euler

$$\sum_i^n f_i = m\dot{\nu}$$

$$\sum_i^n \tau_i = I\dot{\omega}$$

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Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1} + \nu_{NB_2} \times \left(\nu_{NB_2} \times r_C\right) + \dot{\nu}_{NB_2} \times r_C\right)$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = m \cdot r_C \times \left(\dot{\nu}_{NB_1} + \nu_{NB_2} \times \nu_{NB_1}\right) + {}^{N}I_B \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^{N}I_B \cdot \nu_{NB_2}$$

Rigid Body Dynamics

Equations of Motion

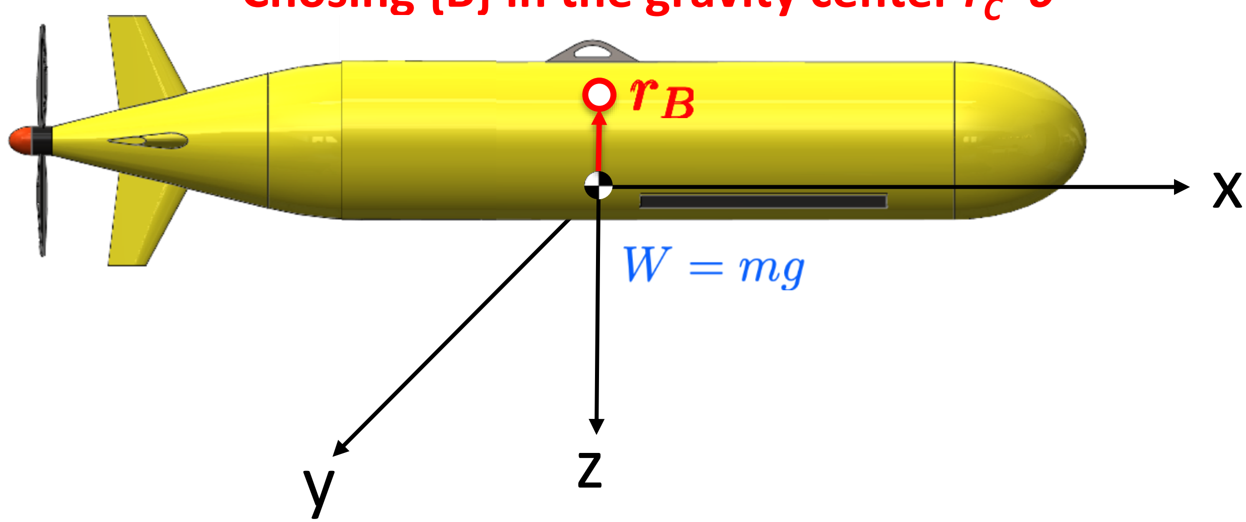
Lineal motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1} + \underbrace{\nu_{NB_2} \times (\nu_{NB_2} \times r_C)}_{\mathbf{0}_{3\times3}} + \underbrace{\dot{\nu}_{NB_2} \times r_C}_{\mathbf{0}_{3\times3}}\right)$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = \underbrace{m \cdot r_C \times (\dot{\nu}_{NB_1} + \nu_{NB_2} \times \nu_{NB_1})}_{\mathbf{0}_{3\times3}} + {}^N I_B \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^N I_B \cdot \nu_{NB_2}$$

Chosing {B} in the gravity center $r_C=0$



Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1}\right.$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = {}^{N}I_B \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^{N}I_B \cdot \nu_{NB_2}$$

Newton-Euler Equations of Motion
With $\boldsymbol{r_C=0}$

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Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m \left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1} \right.$$

Linear Acceleration

Angular motion [Euler Rotation Eq

$$\tau_{B_2} = {}^{N}I_B \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^{N}I$$

Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

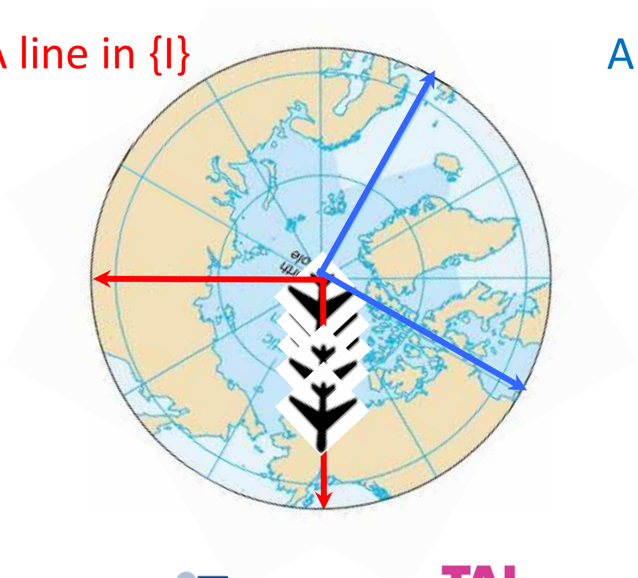
$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1}\right)$$

Coriolis Acceleration

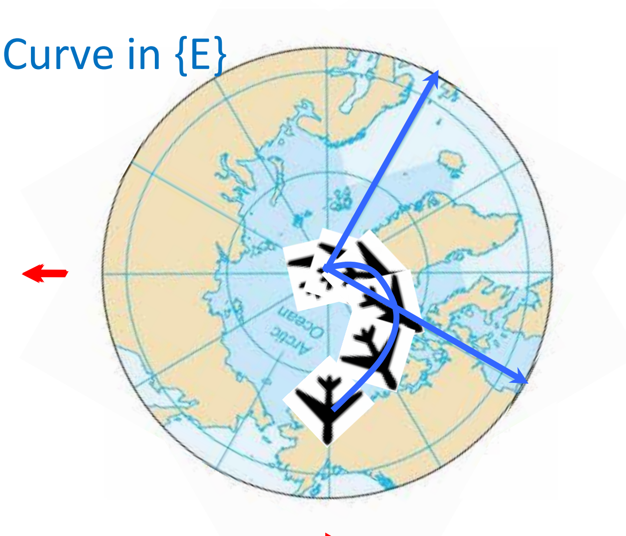
Angular motion [Eule… quation]:

$$\tau_{B_2} = {}^N I_B \cdot \dot{\nu}_{NB_2} + \ldots I_B \cdot \nu_{NB_2}$$

A line in {I}



A Curve in {E}



Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1}\right.$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = {}^{N}I_B \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^{N}I_B \cdot \nu_{NB_2}$$

Angular Acceleration

Newton-Euler Equations of Motion
With $\boldsymbol{r_C = 0}$

Rigid Body Dynamics

Equations of Motion

Lineal motion [Newton 2nd's Law]:

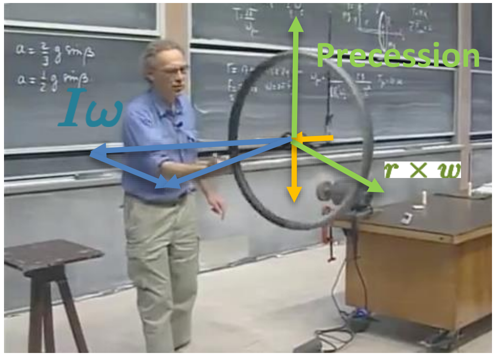
$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1}\right)$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = {}^{N}I_B \cdot \dot{\nu}_{NB_2} + \underbrace{\nu_{NB_2} \times {}^{N}I_B \cdot \nu_{NB_2}}_{\text{Gyroscopic Precession}}$$

Newton-Euler Equations of Motion
With ***r_C=0***

Gyroscopic Precession



Rigid Body Dynamics

Equations of Motion

Linear motion [Newton 2nd's Law]:

$$\tau_1 = m \cdot a_{EC_1} = m\left(\nu_{NB_2} \times \nu_{NB_1} + \dot{\nu}_{NB_1}\right)$$

Angular motion [Euler Rotation Equation]:

$$\tau_{B_2} = {}^{N}I_{B} \cdot \dot{\nu}_{NB_2} + \nu_{NB_2} \times {}^{N}I_{B} \cdot \nu_{NB_2}$$

Matrix Equations of Motion

$$\begin{bmatrix} \boldsymbol{\tau_1} \\ \boldsymbol{\tau_2} \end{bmatrix} = \begin{bmatrix} m\boldsymbol{I_{3\times3}} & \boldsymbol{0_{3\times3}} \\ \boldsymbol{0_{3\times3}} & {}^{N}\boldsymbol{I_B} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\nu}}_{\boldsymbol{NB_1}} \\ \dot{\boldsymbol{\nu}}_{\boldsymbol{NB_2}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0_{3\times3}} & -m[\boldsymbol{\nu_{NB_1}}]_\times \\ \boldsymbol{0_{3\times3}} & -[{}^{N}\boldsymbol{I_B}\boldsymbol{\nu_{NB_2}}]_\times \end{bmatrix} \begin{bmatrix} \boldsymbol{\nu_{NB_1}} \\ \boldsymbol{\nu_{NB_2}} \end{bmatrix}$$

Can be expressed as: $\boldsymbol{\tau_{RB}} = \boldsymbol{M_{RB}}\dot{\boldsymbol{\nu}} + \boldsymbol{C_{RB}}(\boldsymbol{\nu})\boldsymbol{\nu}$

where $\boldsymbol{M_{RB}} = \begin{bmatrix} \boldsymbol{M_{11}} & \boldsymbol{M_{12}} \\ \boldsymbol{M_{21}} & \boldsymbol{M_{22}} \end{bmatrix} = \begin{bmatrix} m\boldsymbol{I_{3\times3}} & -m[\boldsymbol{r_C}]_\times \\ m[\boldsymbol{r_C}]_\times & {}^{N}\boldsymbol{I_B} \end{bmatrix}$

$$\begin{aligned} \boldsymbol{r} &= [x\ y\ z]^T \\ \boldsymbol{v} &= [a\ b\ c]^T \end{aligned} \Rightarrow \boldsymbol{r} \times \boldsymbol{v} = [\boldsymbol{r}]_\times \boldsymbol{v} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -bz + yc \\ za - xc \\ -ya + xb \end{bmatrix}$$

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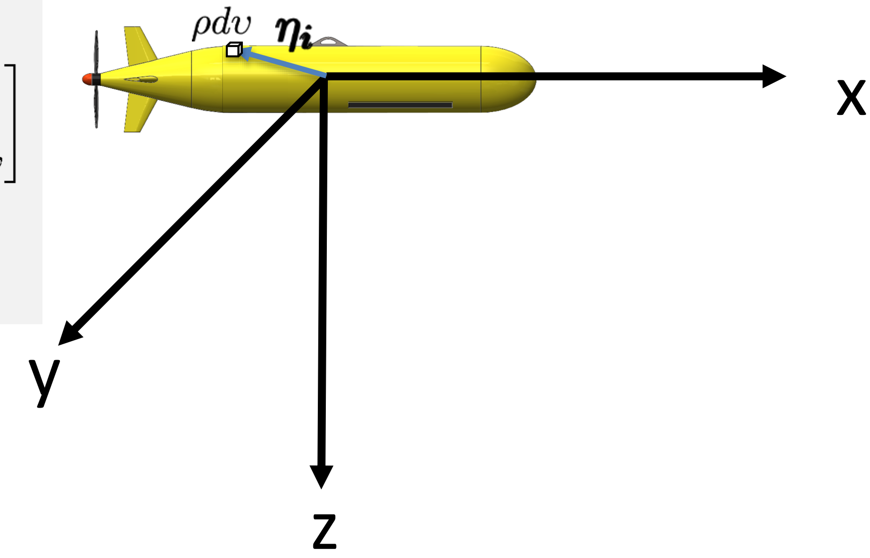
Rigid Body Dynamics

Inertia Tensor

- Inertial tensor describes the mass distribution of the body

$$
\begin{aligned}
{}^{N}\boldsymbol{I_B} &= -\int_v [\boldsymbol{\eta_i}]_\times^2 \cdot \rho \cdot dv = -\int_v \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}^2 \cdot \rho \cdot dv \\
&= \int_v \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{bmatrix} \cdot \rho \cdot dv \\
&= \begin{bmatrix} \int_v (y^2+z^2)\cdot\rho\cdot dv & -\int_v xy\cdot\rho\cdot dv & -\int_v xz\cdot\rho\cdot dv \\ -\int_v yx\cdot\rho\cdot dv & \int_v (x^2+z^2)\cdot\rho\cdot dv & -\int_v yz\cdot\rho\cdot dv \\ -\int_v zx\cdot\rho\cdot dv & -\int_v zy\cdot\rho\cdot dv & \int_v (x^2+y^2)\cdot\rho\cdot dv \end{bmatrix} \\
&= \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}
\end{aligned}
$$

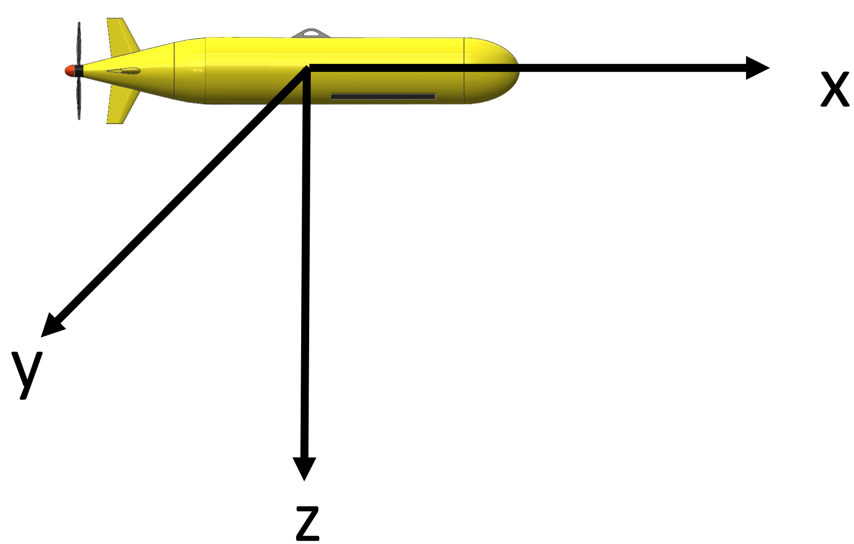


Rigid Body Dynamics

Inertia Tensor

- Inertial products involving an axis orthogonal to a symmetry plane are 0

$$\boldsymbol{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

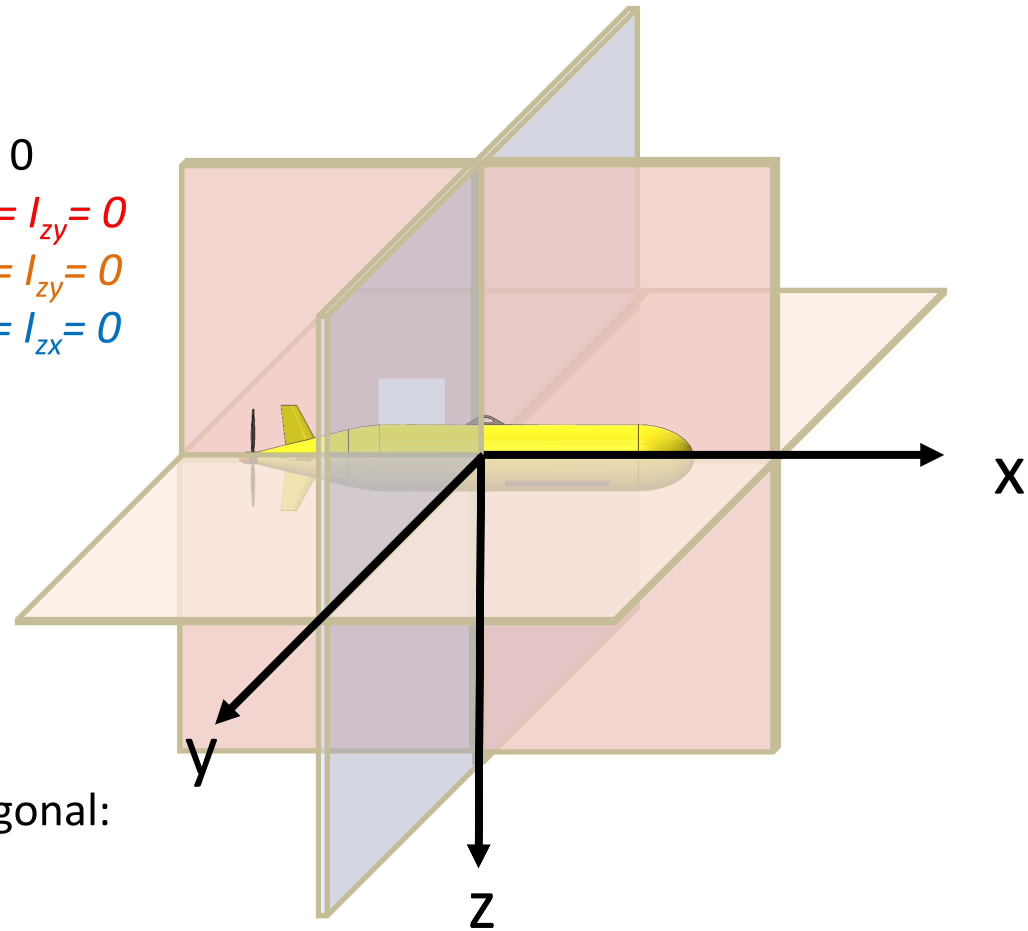


Rigid Body Dynamics

Inertia Tensor

- Inertial products involving an axis orthogonal to a symmetry plane are 0
- XZ Symmetry ⇒ y⊥XZ ⇒ $I_{xy} = I_{yx} = I_{yz} = I_{zy} = 0$
- XY Symmetry ⇒ z⊥XY ⇒ $I_{xz} = I_{yx} = I_{zz} = I_{zy} = 0$
- YZ Symmetry ⇒ x⊥YZ ⇒ $I_{xy} = I_{yx} = I_{xz} = I_{zx} = 0$

$$\boldsymbol{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$



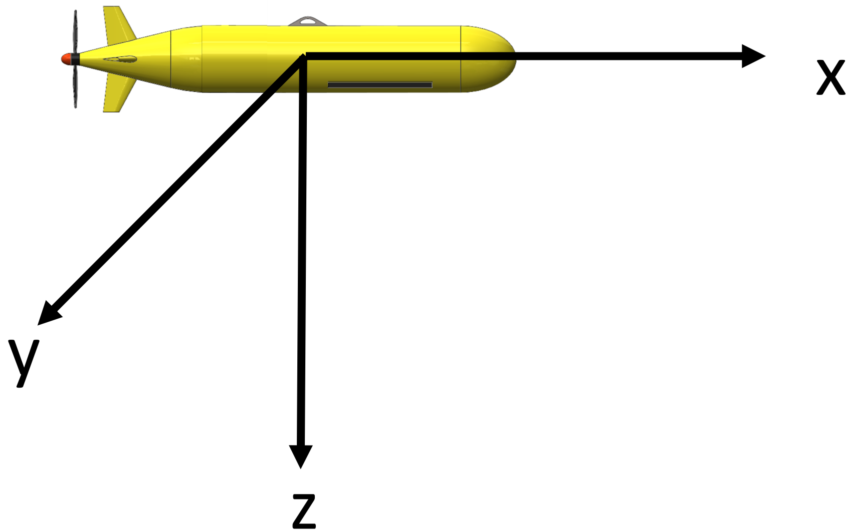
- 2 planes of symmetry means ***I*** is diagonal:

$$\boldsymbol{I} = Diag\{I_{xx}, I_{yy}, I_{zz}\}$$

Hydrodynamic Forces

Added Mass Forces

Pressure-induced forces required to accelerate a certain amount of surrounding water moving with the vehicle. Opposes to the vehicle motion. They depend on the vehicle shape.



Hydrodynamic Forces

Added Mass Forces

Pressure-induced forces required to accelerate a certain amount of surrounding water moving with the vehicle. Opposes to the vehicle motion. They depend on the vehicle shape.

$$\boldsymbol{\tau_a} = -\begin{bmatrix} X_{\dot u} & X_{\dot v} & X_{\dot w} & X_{\dot p} & X_{\dot q} & X_{\dot r} \\ X_{\dot v} & Y_{\dot v} & Y_{\dot w} & Y_{\dot p} & Y_{\dot q} & Y_{\dot r} \\ X_{\dot w} & Y_{\dot w} & Z_{\dot w} & Z_{\dot p} & Z_{\dot q} & Z_{\dot r} \\ X_{\dot p} & Y_{\dot p} & Z_{\dot p} & K_{\dot p} & K_{\dot q} & K_{\dot r} \\ X_{\dot q} & Y_{\dot q} & Z_{\dot q} & K_{\dot q} & M_{\dot q} & M_{\dot r} \\ X_{\dot r} & Y_{\dot r} & Z_{\dot r} & K_{\dot r} & M_{\dot r} & N_{\dot r} \end{bmatrix} \begin{bmatrix} \dot u \\ \dot v \\ \dot w \\ \dot p \\ \dot q \\ \dot r \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

Where:

$$\begin{aligned} a_1 &= X_{\dot u}u + X_{\dot v}v + X_{\dot w}w + X_{\dot p}p + X_{\dot q}q + X_{\dot r}r \\ a_2 &= X_{\dot v}u + Y_{\dot v}v + Y_{\dot w}w + Y_{\dot p}p + Y_{\dot q}q + Y_{\dot r}r \\ a_3 &= X_{\dot w}u + Y_{\dot w}v + Z_{\dot w}w + Z_{\dot p}p + Z_{\dot q}q + Z_{\dot r}r \\ b_1 &= X_{\dot p}u + Y_{\dot p}v + Z_{\dot p}w + K_{\dot p}p + K_{\dot q}q + K_{\dot r}r \\ b_2 &= X_{\dot q}u + Y_{\dot q}v + Z_{\dot q}w + K_{\dot q}p + M_{\dot q}q + M_{\dot r}r \\ b_3 &= X_{\dot r}u + Y_{\dot r}v + Z_{\dot r}w + K_{\dot r}p + M_{\dot r}q + N_{\dot r}r \end{aligned}$$

$$\boldsymbol{M_A} = \begin{bmatrix} \boldsymbol{A_{11}} & \boldsymbol{A_{12}} \\ \boldsymbol{A_{21}} & \boldsymbol{A_{22}} \end{bmatrix}$$

Hydrodynamic Forces

Added Mass Forces

Pressure-induced forces required to accelerate a certain amount of surrounding water moving with the vehicle. Opposes to the vehicle motion. They depend on the vehicle shape.

Can be written using a matrix equation:

$$\boldsymbol{\tau_A} = \boldsymbol{M_A}\dot{\boldsymbol{\nu}} + \boldsymbol{C_A}(\boldsymbol{\nu})\boldsymbol{\nu}$$

Where:

$$M_A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \; C_A = \begin{bmatrix} 0_{3\times3} & -[A_{11}\nu_1 + A_{12}\nu_2]_\times \\ -[A_{11}\nu_1 + A_{12}\nu_2]_\times & -[A_{21}\nu_1 + A_{22}\nu_2]_\times \end{bmatrix}$$

A common simplification is to consider M_A Diagonal

$$\boldsymbol{M_A} = Diag\{X_{\dot{u}} \; Y_{\dot{v}} \; Z_{\dot{w}} \; K_{\dot{p}} \; M_{\dot{q}} \; N_{\dot{r}}\}$$

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Hydrodynamic Forces

Added Mass Forces

$$\boldsymbol{M_A} = Diag\{X_{\dot{u}}\ Y_{\dot{v}}\ Z_{\dot{w}}\ K_{\dot{p}}\ M_{\dot{q}}\ N_{\dot{r}}\}$$

$$X_{\dot{u}} = -\frac{\alpha_0}{2-\alpha_0}m$$

$$Y_{\dot{v}} = -\frac{\beta_0}{2-\beta_0}m$$

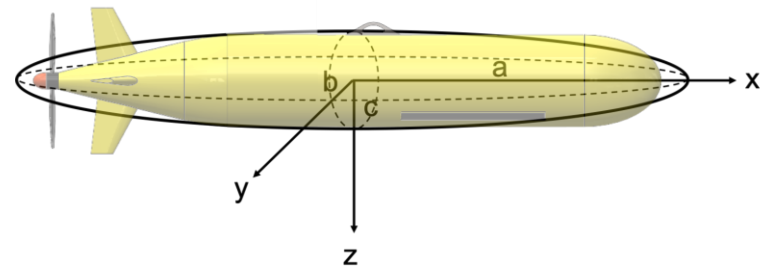
$$Z_{\dot{w}} = 0$$

$$K_{\dot{p}} = M_{\dot{q}} = -\frac{1}{5}\frac{(b^2-a^2)^2(\alpha_0-\beta_0)}{2(b^2-a^2)+(b^2+a^2)(\beta_0-\alpha_0)}m$$

$$m = \frac{4}{3}\pi a b^2$$

$$e^2 = 1 - \left(\frac{b}{a}\right)^2$$

$$\alpha_0 = \frac{2(1-e^2)}{e^3}\left(\frac{1}{2}ln\left(\frac{1+e}{1-e}\right) - e\right)$$



Hydrodynamic Forces

Restoring Forces

Forces acting on the submerged body trying to bring it to an equilibrium point:

Gravity Force

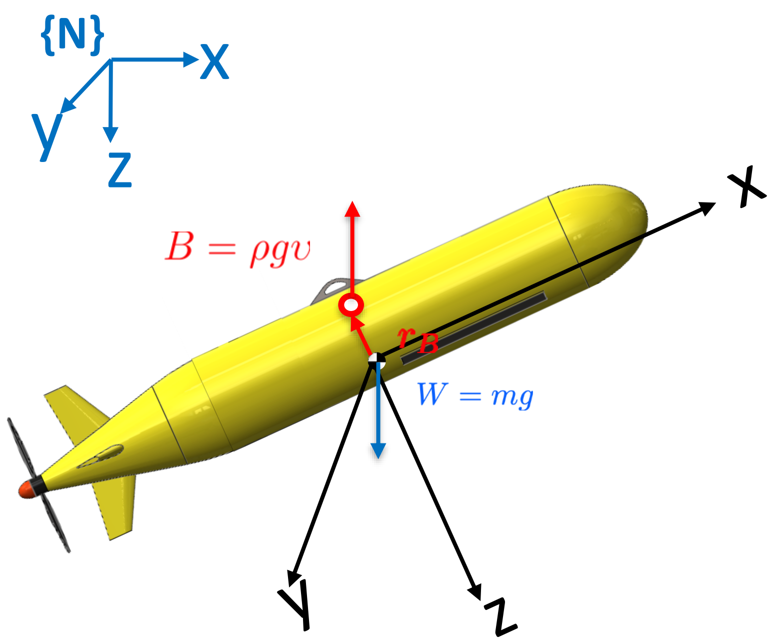
$$^{N}\tau_{g_1} = m \cdot g$$

$$\tau_g = \begin{bmatrix} {}^{B}R(\eta)_N \cdot {}^{N}\tau_{g_1} \\ {}^{B}R(\eta)_N \cdot r_C \times {}^{N}\tau_{g_1} \end{bmatrix}$$

Buoyancy Force

$$^{N}\tau_{b_1} = -g\rho\upsilon$$

$$\tau_b = \begin{bmatrix} {}^{B}R(\eta)_N \cdot {}^{N}\tau_{b_1} \\ {}^{B}R(\eta)_N \cdot r_C \times {}^{N}\tau_{b_1} \end{bmatrix}$$



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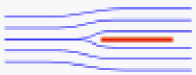
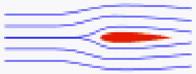
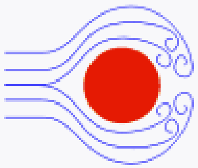
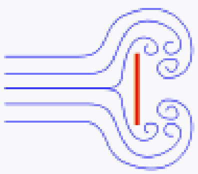
Hydrodynamic Forces

Damping Forces

Skin Friction: Linear Friction due to the laminar boundary layer.
Form Drag: Quadratic non-linear friction due to the turbulent boundary layer.

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%

Can be written using a matrix equation:

$$\boldsymbol{\tau}_{\boldsymbol{D}}^{\boldsymbol{T}} = -\boldsymbol{\tau}_{\boldsymbol{D}_{\boldsymbol{\nu}}}^{\boldsymbol{T}} \cdot \boldsymbol{\nu} - \boldsymbol{\tau}_{\boldsymbol{D}_{\boldsymbol{\nu}|\boldsymbol{\nu}|}}^{\boldsymbol{T}} \cdot \boldsymbol{\nu} \cdot |\boldsymbol{\nu}|$$

Where:

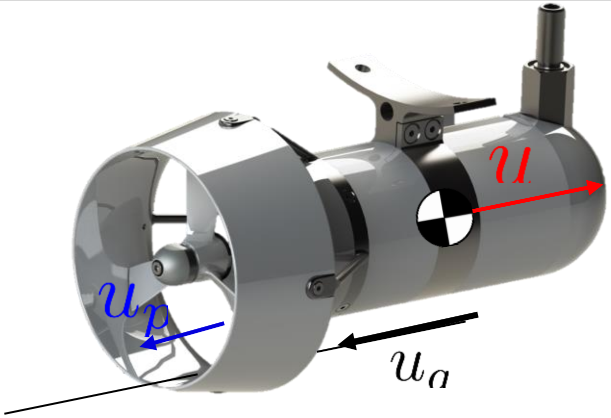
$$\boldsymbol{\tau}_{\boldsymbol{D}_{\boldsymbol{\nu}}} = diag\{X_u \ Y_v \ Z_w \ K_p \ M_q \ N_r\}$$

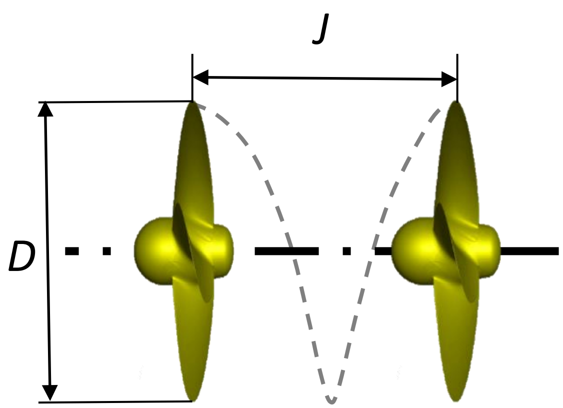
$$\boldsymbol{\tau}_{\boldsymbol{D}_{\boldsymbol{\nu}|\boldsymbol{\nu}|}} = diag\{X_{u|u|} \ Y_{v|v|} \ Z_{w|w|} \ K_{p|p|} \ M_{q|q|} \ N_{r|r|}\}$$

Propulsion

Thruster Forces

When the propeller rotates at *n* [rev/s] it exerts a thrust *T* and a torque *Q.*





$u[m/s]$	Surge speed
$n[rev/s]$	Propeller angular speed
$T[N]$	Thrust
$Q[Nm]$	Torque
η_p	Efficiency
$V_a[m/s]$	Advance Speed (fluid velocity at the propeller when it is at rest)
$J = \frac{V_a}{nD}$	Advance Ratio. Distance travelled in one propeller revolution

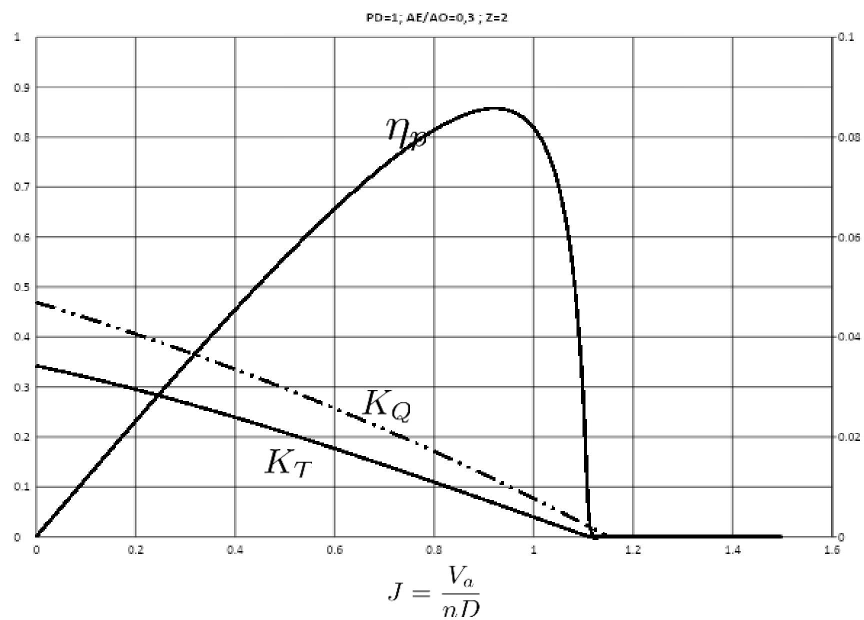
Propulsion

Thruster Forces

When the propeller rotates at *n* [rev/s] it exerts a thrust *T* and a torque *Q.*

$$T = \rho D^4 K_T(J) n|n|$$

- K_T is ≈ linear in J
  $$K_T = \alpha_1 J + \alpha_2$$
- So T becomes
  $$T = \rho D^4 (\alpha_1 \frac{V_a}{nD} + \alpha_2) n|n|$$
  $$= \rho D^4 \alpha_2 n|n| + \rho D^3 \alpha_1 |n| V_a$$
- Obtaining a bilinear model
  $$T = T_{n|n|} n|n| - T_{|n|V_a} |n| V_a$$
  $$T_{n|n|} = \rho D^4 \alpha_2$$
  $$T_{|n|V_a} = \rho D^3 \alpha_1$$



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Propulsion

Thruster Forces

When the propeller rotates at *n* [rev/s] it exerts a thrust *T* and a torque *Q*.

$$Q = \rho D^5 K_Q(J) n|n|$$

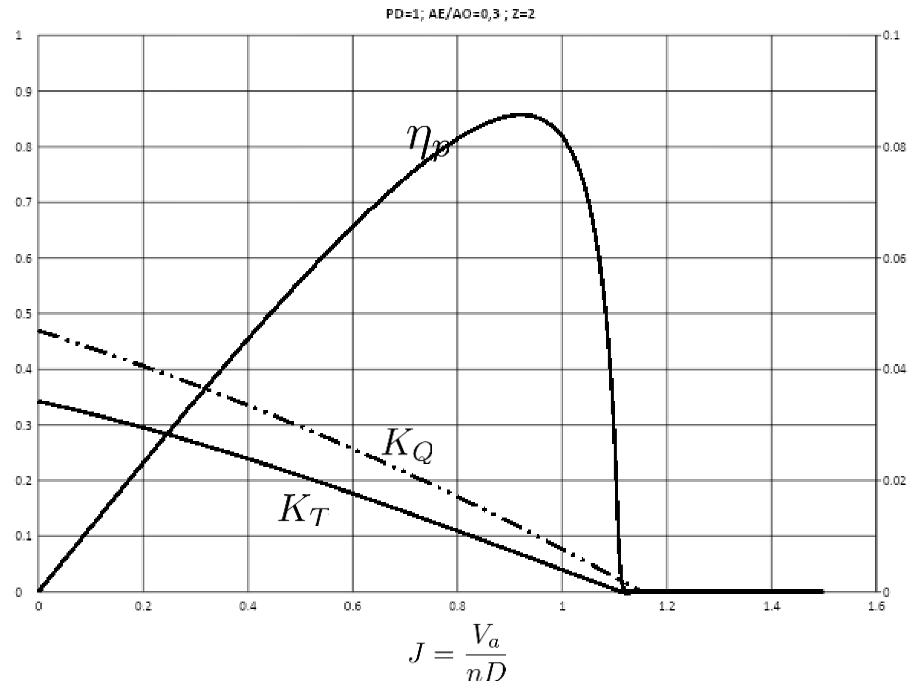
- K_Q is ≈ linear in J

$$K_Q = \beta_1 J + \beta_2$$

- Obtaining a bilinear model

$$Q = Q_{n|n|} n|n| - Q_{|n|V_a} |n| V_a$$

$$Q_{n|n|} = \rho D^4 \beta_2$$
$$Q_{|n|V_a} = \rho D^3 \beta_1$$



Propulsion

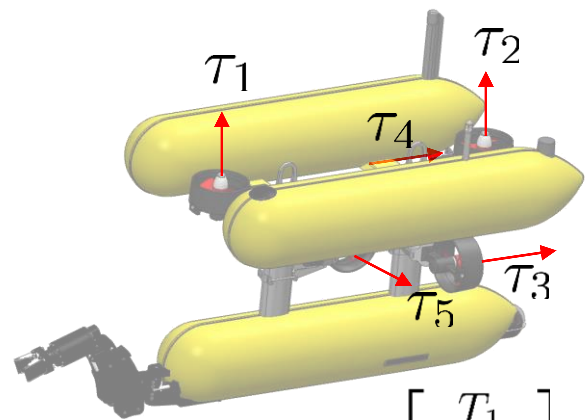
Thruster Forces

When the propeller rotates at *n* [rev/s] it exerts a thrust *T* and a torque *Q.*

- Resultant Force

Unitary vector in the Thrust direction

$$\boldsymbol{\tau_1} = \sum_1^m \boldsymbol{\tau_{m i_1}} = \sum_1^m \boldsymbol{\delta_{T_i}} T_i$$



$$\boldsymbol{\tau_2} = \sum_1^m \boldsymbol{r_i} \times \boldsymbol{\tau_{m i_1}} + \boldsymbol{\tau_{m i_2}} = \sum_1^m \boldsymbol{r_i} \times \boldsymbol{\delta_{T_i}} T_i + \boldsymbol{\delta_{Q_i}} Q_i$$

Force application point

$$\begin{bmatrix} \boldsymbol{\tau_1} \\ \boldsymbol{\tau_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta_{T_1}} & \boldsymbol{\delta_{T_2}} & \dots & \boldsymbol{\delta_{T_m}} & \boldsymbol{0_{3\times 1}} & \boldsymbol{0_{3\times 1}} & \dots & \boldsymbol{0_{3\times 1}} \\ \boldsymbol{r_1} \times \boldsymbol{\delta_{T_1}} & \boldsymbol{r_2} \times \boldsymbol{\delta_{T_2}} & \dots & \boldsymbol{r_m} \times \boldsymbol{\delta_{T_m}} & \boldsymbol{\delta_{Q_1}} & \boldsymbol{\delta_{Q_2}} & \dots & \boldsymbol{\delta_{Q_m}} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_m \\ Q_1 \\ Q_2 \\ \vdots \\ Q_m \end{bmatrix}$$

Thruster Configuration Matrix

Environmental Forces

Ocean currents

- A common approach considers only irrotational currents constant in the N-Frame:

$$ {}^{N}\boldsymbol{\nu_c} = [{}^{N}u_c \; {}^{N}v_c \; {}^{N}w_c \; 0 \; 0 \; 0]^T $$

- They can be referenced to the B-Frame:

$$ \boldsymbol{\nu_c} = \boldsymbol{J(\eta)}^{\boldsymbol{N}}\boldsymbol{\nu_c}. $$

- Now we can define the fluid relative velocity:

$$ \boldsymbol{\nu_r} = \boldsymbol{\nu} - \boldsymbol{\nu_c}. $$

The Model

Hydrodynamic Model

The complete model includes the rigid body dynamics as well as the hydrodynamics

$$\tau = M_{RB}\dot{\nu} + C_{RB}(\nu)\nu + M_A\dot{\nu} + C_A(\nu)\nu + D_\nu \nu + D_{\nu|\nu|}(\nu)\nu + G(\eta)$$

Force Torque — Rigid Body Dynamics — Hydrodynamics — Restoring Forces

The Model

Hydrodynamic Model

The complete model includes the rigid body dynamics as well as the hydrodynamics

$$\tau = M_{RB}\dot{\nu} + C_{RB}(\nu)\nu + M_A\dot{\nu} + C_A(\nu)\nu + D_\nu \nu + D_{\nu|\nu|}(\nu)\nu + G(\eta)$$

$$M = M_{RB} + M_A \qquad C = C_{RB} + C_A \qquad D = D_\nu + D_{\nu|\nu|}$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + G(\eta) = \tau_{thr} + \tau_{fins} + \tau_{ext}$$

- Taking into account the currents

:

$$M\dot{\nu}_r + C(\nu_r)\nu_r + D(\nu_r)\nu_r + G(\eta) = \tau_{thr} + \tau_{fins} + \tau_{ext}$$

The Model

Common Simplifications

- The B-Frame is located at the gravity center ($\mathbf{r_C} = \mathbf{0}$).
- The products of inertia are negligible so $\boldsymbol{I} = diag\{I_{xx}, I_{yy}, I_{zz}\}$. This happens when the vehicle has 3 planes of symmetry.
- The added mass matrix and the damping matrices can be considered diagonal.

$$
\begin{bmatrix} X \\ Y \\ Z \\ K \\ M \\ N \end{bmatrix} = \left(\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} - \begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix} \right) \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}
$$

$$
+ \left(\begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_{zz}r & -I_{yy}q \\ 0 & 0 & 0 & -I_{zz}r & 0 & I_{xx}p \\ 0 & 0 & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \right) \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}
$$

$$
+ \left(\begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} + \begin{bmatrix} X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{|v\ v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{|w|w}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{|p|p}|p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{|q|q}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{r|r}|r| \end{bmatrix} \right) \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}
$$

$$
+ \begin{bmatrix} (W-B)sin(\theta) \\ -(W-B)cos(\theta)sin(\phi) \\ -(W-B)cos(\theta)cos(\phi) \\ By_b cos(\theta)cos(\phi) \quad Bz_b cos(\theta)sin(\phi) \\ -Bz_b sin(\theta) - Bx_b cos(\theta)cos(\phi) \\ By_b sin(\theta) + Bx_b cos(\theta)sin(\phi) \end{bmatrix} \qquad (1)
$$

Model Identification

- How do we estimate the 27 parameters of the model?

$$
\begin{bmatrix} X \\ Y \\ Z \\ K \\ M \\ N \end{bmatrix} = \left(\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} - \begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix} \right) \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}
$$

$$
+ \left(\begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_{zz}r & -I_{yy}q \\ 0 & 0 & 0 & -I_{zz}r & 0 & I_{xx}p \\ 0 & 0 & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \right) \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}
$$

$$
+ \left(\begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} + \begin{bmatrix} X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{|v|v}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{|w|w}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{|p|p}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{|q|q}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{r|r|}|r| \end{bmatrix} \right) \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}
$$

$$
+ \begin{bmatrix} (W - B)sin(\theta) \\ -(W - B)cos(\theta)sin(\phi) \\ -(W - B)cos(\theta)cos(\phi) \\ By_b cos(\theta)cos(\phi) - Bz_b cos(\theta)sin(\phi) \\ -Bz_b sin(\theta) - Bx_b cos(\theta)cos(\phi) \\ By_b sin(\theta) + Bx_b cos(\theta)sin(\phi) \end{bmatrix} \quad (1)
$$

Model Identification

- Let us consider the surge equation of motion:

$$X + (sin\theta B - sin\theta W) - (X_u + X_{u|u|}|u|)u + \tau_p = (m - X_{\dot{u}})\dot{u} + [(m - Z_{\dot{w}})wq] - [(m - Y_{\dot{p}})vr]$$

Thruster Forces | Buoyancy & gravity | Skin Friction | Perturbation

Resultant Force in x

- If Neutrally Buoyant ⇒ W=B
- If the robot performs and a single DOF uncoupled motion ⇒ *B=W* & *w=q=v=r=0*

$$X - (X_u + X_{u|u|}|u|)u + \tau_p = (m - X_{\dot{u}})\dot{u},$$

$$\dot{u} = \underbrace{\frac{X}{m - X_{\dot{u}}}}_{\delta_u} - \underbrace{\frac{X_u}{m - X_{\dot{u}}}}_{\alpha_u}u - \underbrace{\frac{X_{u|u|}|u|}{m - X_{\dot{u}}}}_{\beta_u}u + \underbrace{\frac{\tau_p}{m - X_{\dot{u}}}}_{\gamma_u}.$$

- In general, this holds for any DOF *i:*

$$\dot{\nu}_i = \alpha_i \nu_i + \beta_i \nu_i |\nu_i| + \gamma_i \tau_i + \delta_i$$

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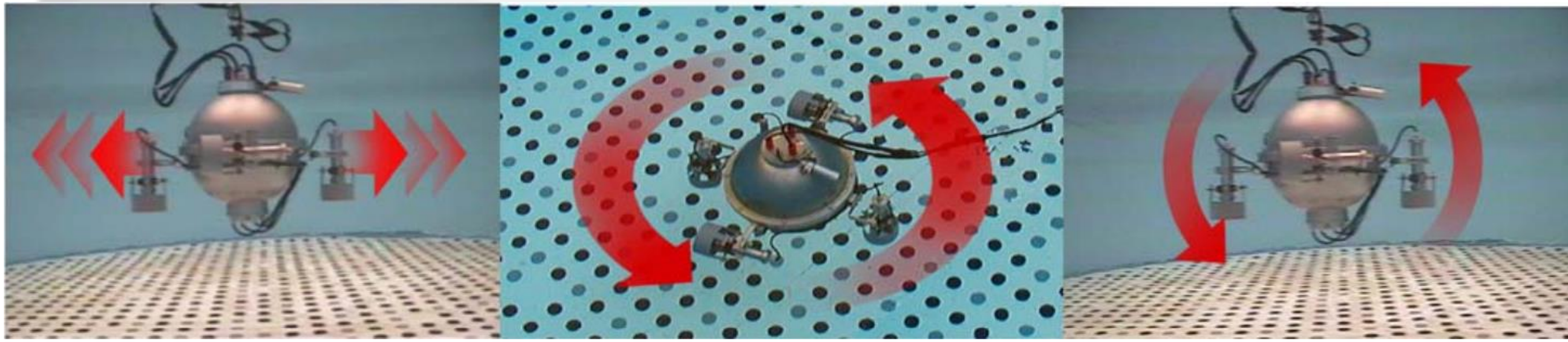
Model Identification - URIS UUV



- An uncoupled experiment is run exciting a single DOF *i*, so the equation of motion is:

$$\dot{\nu}_i = \alpha_i \nu_i + \beta_i \nu_i |\nu_i| + \gamma_i \tau_i + \delta_i$$

- The force position and velocity times series are measured, being used in the following equation which is linear in the set of model parameters

$$\underbrace{\begin{bmatrix} \dot{\nu}_{i_1} \\ \dot{\nu}_{i_2} \\ \vdots \\ \dot{\nu}_{i_N} \end{bmatrix}}_{\nu} = \underbrace{\begin{bmatrix} \nu_{i_1} & \nu_{i_1}|\nu_{i_1}| & \tau_{i_1} & \eta_1 \\ \nu_{i_2} & \nu_{i_2}|\nu_{i_2}| & \tau_{i_2} & \eta_2 \\ \vdots & \vdots & \vdots & \vdots \\ \nu_{i_N} & \nu_{i_N}|\nu_{i_N}| & \tau_{i_N} & \eta_N \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} \alpha_i \\ \beta_i \\ \gamma_i \\ \delta_i \end{bmatrix}}_{\theta_{LS}} + \underbrace{\begin{bmatrix} \varepsilon_{i_1} \\ \varepsilon_{i_2} \\ \vdots \\ \varepsilon_{i_N} \end{bmatrix}}_{\varepsilon}$$

- The parameters are estimated through Least-Squares algorithm

$$\hat{\theta}_{LS} = (H^T H)^{-1} H^T y$$
$$P_{LS} = (H^T H)^{-1}$$

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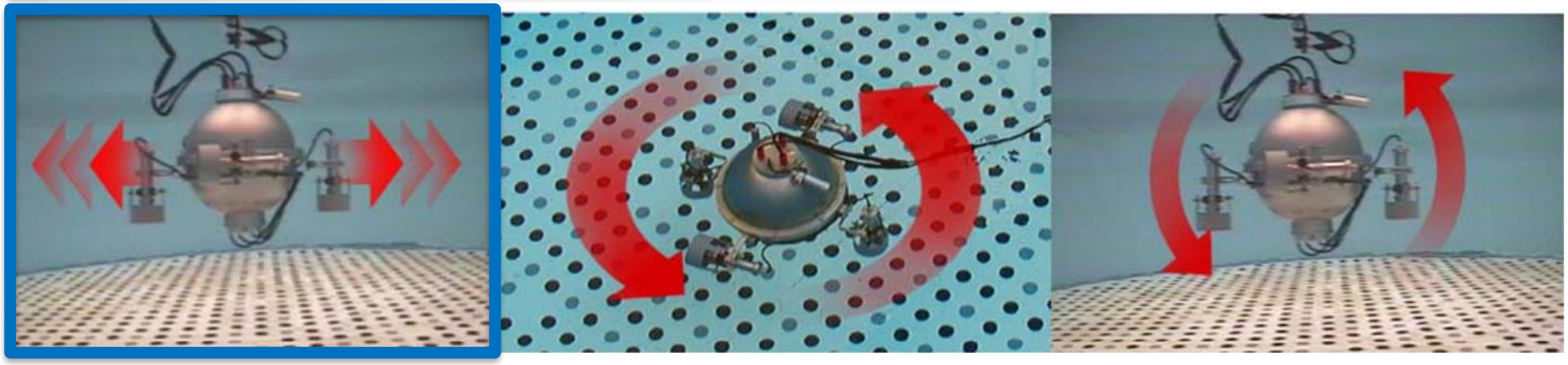
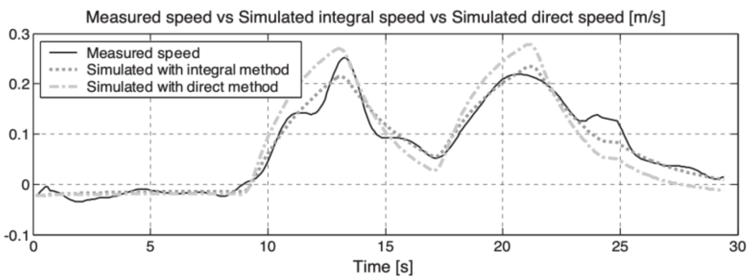
Model Identification - URIS UUV





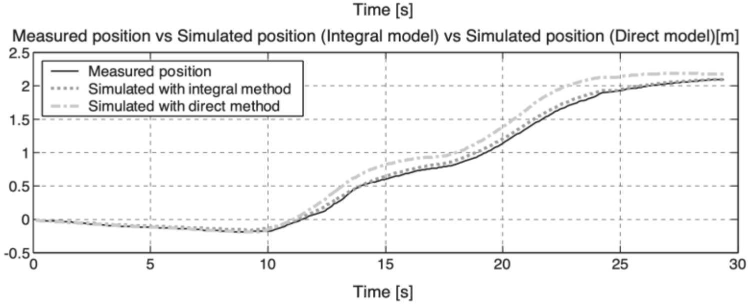
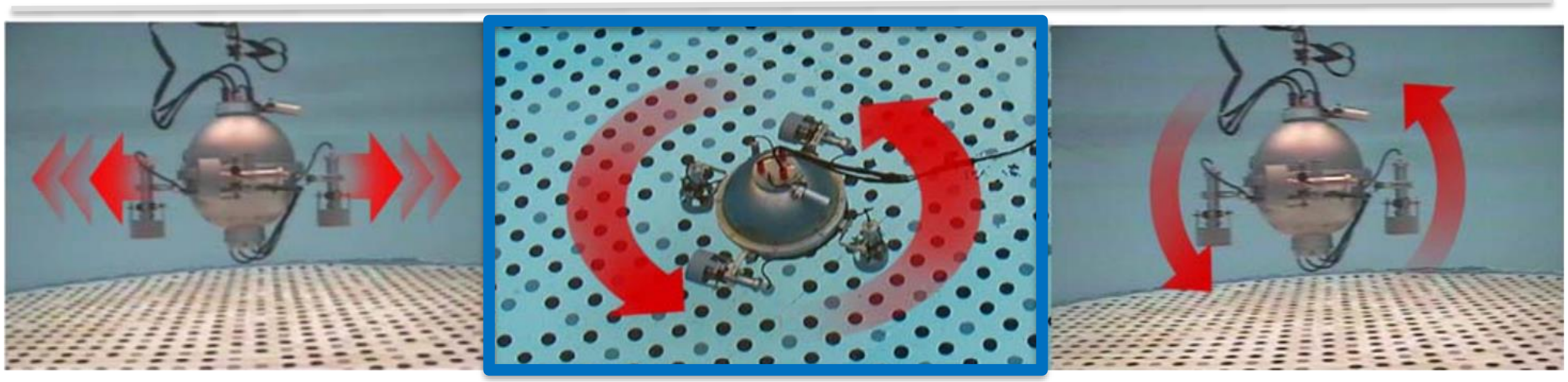


Table 1: Identification results for the surge experiment

Experiment		α_i	γ_i	δ_i	J_i
1	$\hat{\theta}_1$	0.4147	0.0236	−0.0010	$1.8432e-4$
	σ_1	0.0025	0.0001	0.0002	
2	$\hat{\theta}_2$	0.4790	0.0321	−0.0090	$2.5973e-4$
	σ_2	0.0022	0.0001	0.0002	
3	$\hat{\theta}_3$	0.5153	0.0295	0.0014	$2.4150e-4$
	σ_3	0.0021	0.0001	0.0002	
Mean	$\hat{\theta}_\mu$	0.4697	0.0284	−0.0028	$2.28517e-4$
	σ_μ	0.00227	0.0001	0.0002	

Model Identification - URIS UUV



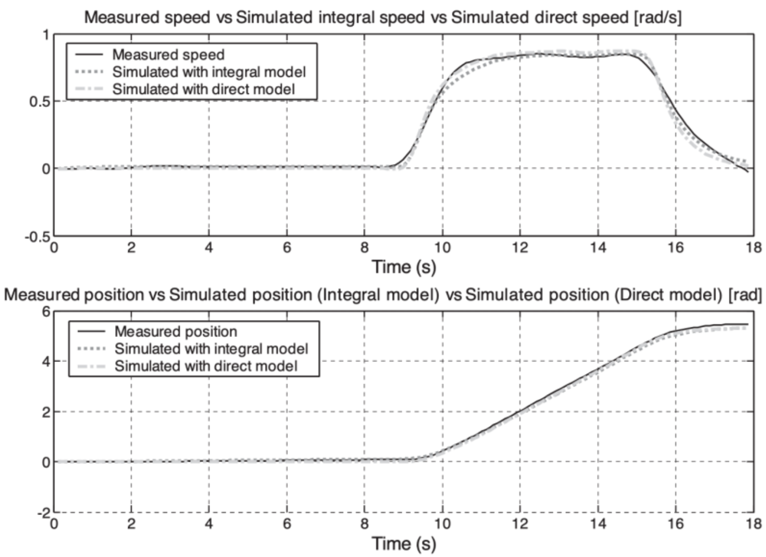


Table 1: Identification results for the yaw experiment

Experiment		α_i	γ_i	δ_i	J_i
1	$\hat{\theta}_1$	1.3755	0.7564	−0.0964	9.7534e − 4
	σ_1	0.0052	0.0026	0.0010	
2	$\hat{\theta}_2$	1.1785	0.4549	−0.4069	0.0033
	σ_2	0.0082	0.0021	0.0035	
3	$\hat{\theta}_3$	1.1279	0.4936	0.2892	0.0032
	σ_3	0.0109	0.0036	0.0041	
4	$\hat{\theta}_4$	1.7541	0.5038	−0.9643	0.0082
	σ_4	0.0216	0.0058	0.0084	
Mean	$\hat{\theta}_\mu$	1.3590	0.5522	−0.2946	0.0039
	σ_μ	0.0114	0.0035	0.0043	

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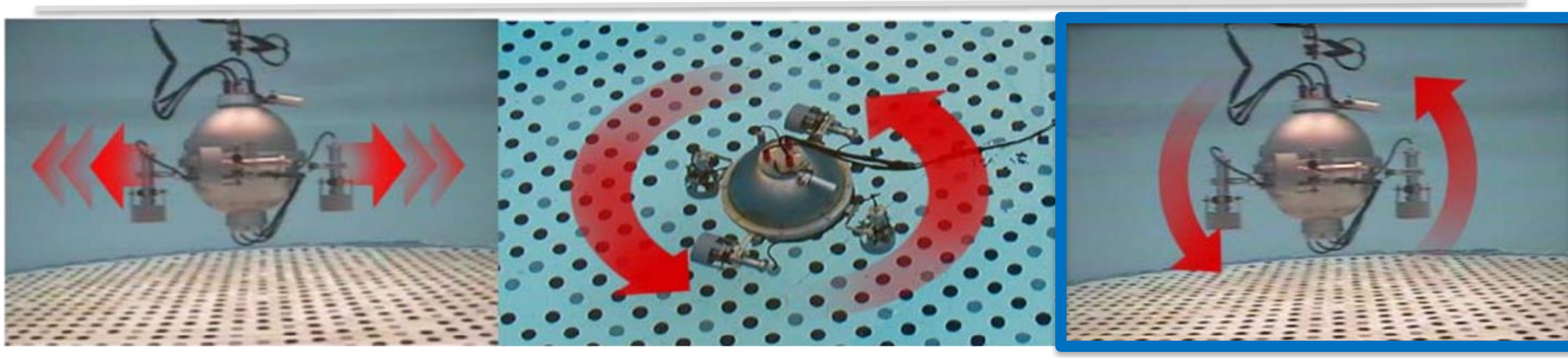
Model Identification - URIS UUV



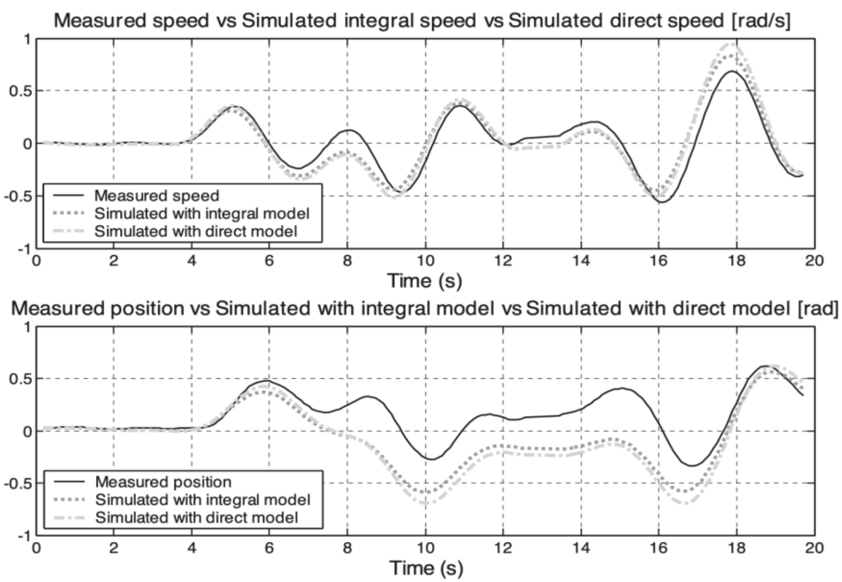


Table 1: Identification results for the pitch experiment

Experiment		α_i	γ_i	J_i
1	$\hat{\theta}_1$	0.5783	1.2121	$0.3260e-4$
	σ_1	0.0027	0.0017	
2	$\hat{\theta}_2$	0.6122	1.0417	$6.7892e-4$
	σ_2	0.0039	0.0024	
3	$\hat{\theta}_3$	0.7092	1.4555	$9.0143e-4$
	σ_3	0.0037	0.0026	
$Mean$	$\hat{\theta}_\mu$	0.6332	1.2364	$8.3765e-4$
	σ_μ	0.0034	0.0022	

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Model Identification - URIS UUV

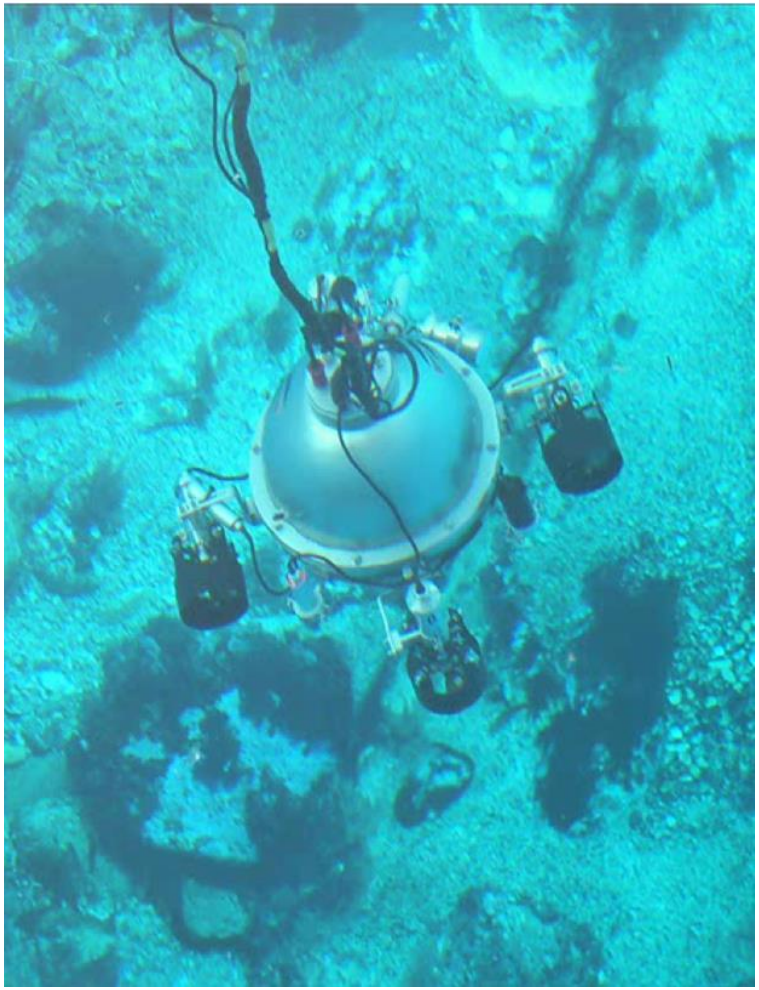

Table 1: URIS UUV hydrodynamical model coefficients

Buoyancy & Weight			Thruster		
W	294.1995	$[N]$	$T_{n\|n\|_{v_f}}$	0.0000143	$[N/rpm^2]$
B	5	$[N]$	$T_{n\|n\|_{v_b}}$	0.0000148	$[N/rpm^2]$
x_b	0.0	$[m]$	$T_{n\|n\|_{h_f}}$	0.0000129	$[N/rpm^2]$
y_b	0.0	$[m]$	$T_{n\|n\|_{h_b}}$	0.0000125	$[N/rpm^2]$
z_b	-0.03	$[m]$			
Mass & Inertia			**Added Mass & Inertia**		
m	30	$[kg]$	$X_{\dot{u}}$	5.2112	$[kg]$
			$Y_{\dot{v}}$	$\approx X_{\dot{u}}$	$[kg]$
			$Z_{\dot{w}}$	$\approx X_{\dot{u}}$	$[kg]$
I_{xx}	0.3468	$[kgm^2]$	$K_{\dot{p}}$	$\approx M_{\dot{q}}$	$[kgm^2]$
I_{yy}	0.3468	$[kgm^2]$	$M_{\dot{q}}$	0.46200	$[kgm^2]$
I_{zz}	0.3468	$[kgm^2]$	$N_{\dot{r}}$	1.46414	$[kgm^2]$
Lineal Damping			**Quadratic Damping**		
X_u	16.53873	$[\frac{Ns}{m}]$	$X_{u\|u\|}$	0	$[\frac{Ns^2}{m^2}]$
Y_v	$\approx X_u$	$[\frac{Ns}{m}]$	$Y_{v\|v\|}$	0	$[\frac{Ns^2}{m^2}]$
Z_w	$\approx X_u$	$[\frac{Ns}{m}]$	$Z_{w\|w\|}$	0	$[\frac{Ns^2}{m^2}]$
K_p	0.51213	$[\frac{Ns}{m}]$	$K_{p\|p\|}$	0	$[\frac{Ns^2}{m^2}]$
M_q	$\approx N_r$	$[\frac{Ns}{m}]$	$M_{q\|q\|}$	0	$[\frac{Ns^2}{m^2}]$
N_r	2.46106	$[\frac{Ns}{m}]$	$N_{r\|r\|}$	0	$[\frac{Ns^2}{m^2}]$
Control Actuators					
Thruster Position			**Thruster Direction**		
r_L	$[0\ -X\ 0]^T$	$[m]$	δ_{T_L}	$[1\ 0\ 0]^T$	*
r_R	$[0\ X\ 0]^T$	$[m]$	δ_{T_R}	$[1\ 0\ 0]^T$	*
r_f	$[X\ 0\ 0]^T$	$[m]$	δ_{T_f}	$[0\ 0\ -1]^T$	*
r_b	$[-X\ 0\ 0]^T$	$[m]$	δ_{T_b}	$[0\ 0\ -1]^T$	*

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Questions ?