Marine Systems & Robotics

The Quest for Provable Robotic Motion Planning

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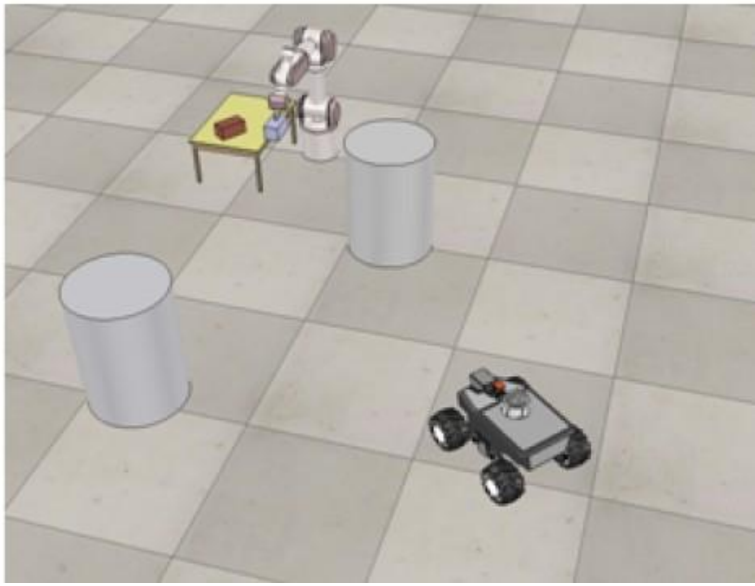
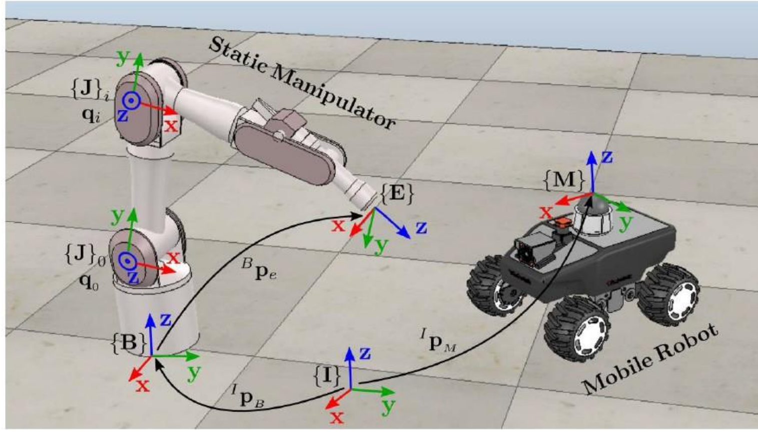
About Motion Planning

- Robotics and Autonomous Systems address the automation of mechanical systems that have sensing, actuation, and computation capabilities.
- A fundamental need in those fields is to convert high-level specifications of tasks from "*Intelligent Entities*" (e.g. humans, but not exclusively) into low-level descriptions of **how to move**.
- **Motion planning** is often used to describe these kinds of problems.

Motion Planning / Control	Originally…	… Recently
Robotics	How to move from one point to another without hitting anything.	…include "complexities" such as uncertainties, multiple bodies, and dynamics
AI	Search for a sequence of logical operators or actions that transform an initial world state into a desired goal state	…extended to include many decision-theoretic (Markov decision processes, imperfect state information, game-theoretic equilibria, Learning, etc.)
Control Theory	Stability, feedback, and optimality	… find feasible, open-loop trajectories for (nonlinear) systems

Planning or Control?



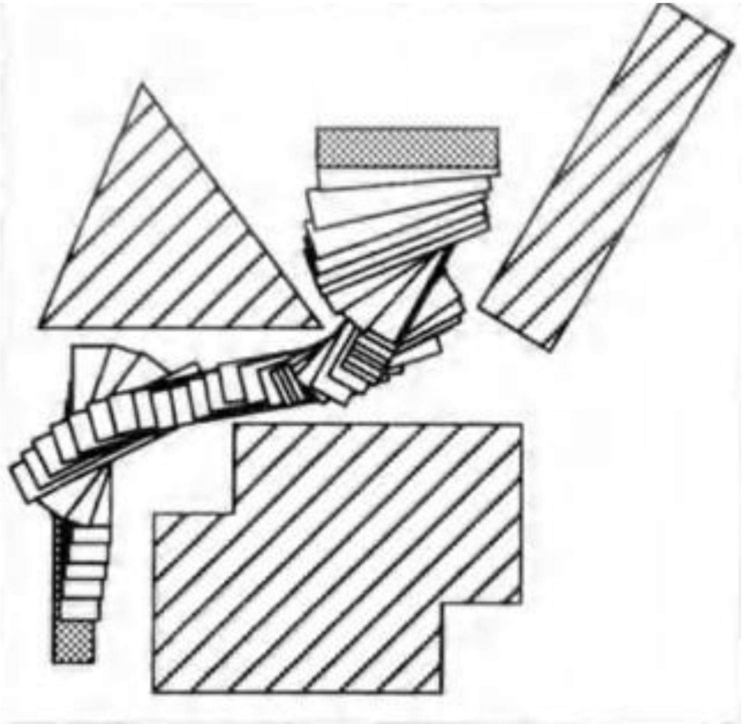


- <u>Robotics</u>: design algorithms that generate appropriate motions by processing complicated geometric models.
- <u>AI</u>: design (decision-theory based) systems to compute appropriate actions.
- <u>Control Theory</u>: design algorithms that compute feasible trajectories through feedback and/or optimality.

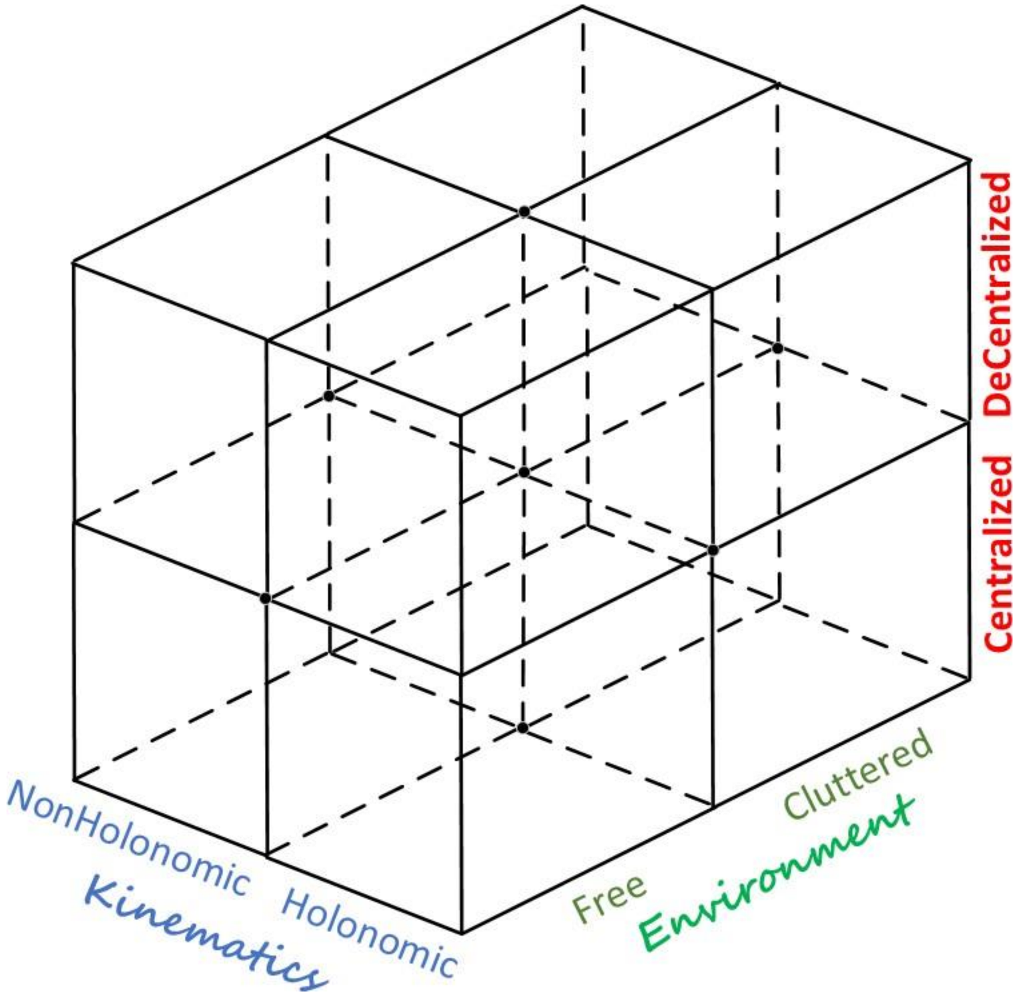
- "Planning" is usually considered as a higher level process than "Control". In our work we do not make such distinctions but we rather consider it in a unified way to address decision making, with no associated notion of "high" or "low" level.

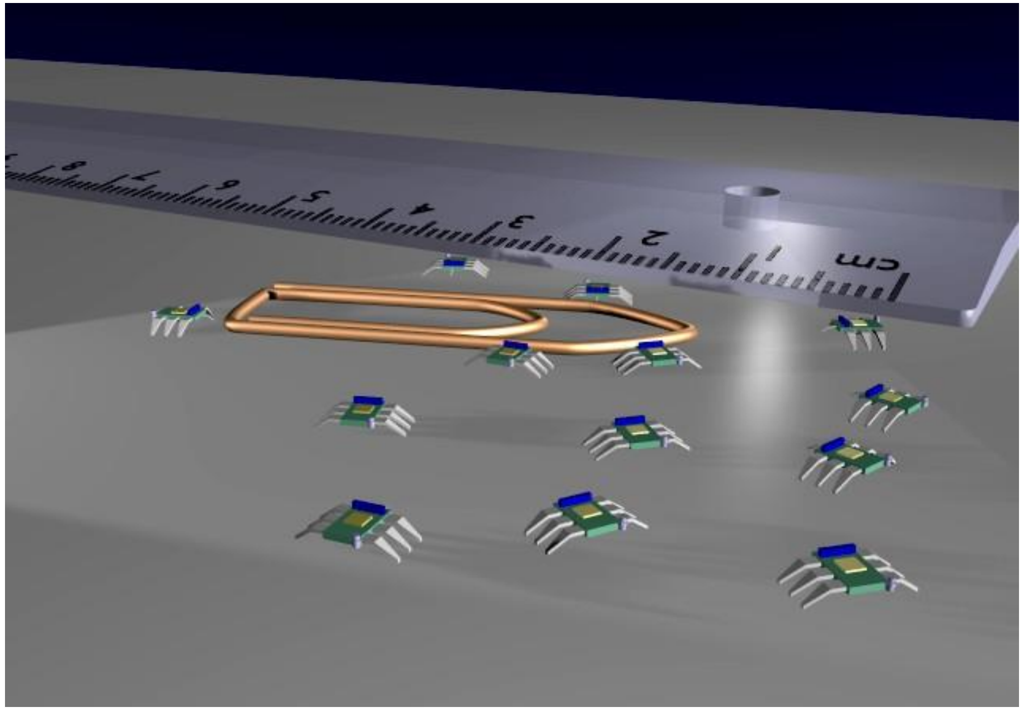
Motion Planning: The Single Agent case

Kinematics / Environment	HOLONOMIC	Non-HOLONOMIC
Free		
Cluttered		



Motion Planning: The Multi - Agent case





Motion Planning: The Single Agent case

Kinematics / Environment	HOLONOMIC	Non-HOLONOMIC
Free	D. Kodistchek & E. Rimon	
Cluttered		

Single Agent: Holonomic Motion Control in Cluttered Environments

Motion Planning

Combinatorial
Canny 1987 (PhD Thesis)

Sampling-Based: RRT, PRM, RRT*
LaValle and Kuffner 2001 (IJRR)
Kavraki et all. 1996 (TRA)
Karaman and Frazzoli 2011 (IJRR)

Implicit
Khatib 1986 (IJRR)
Lumelsky & Stepanov 1987 (Algorithmica)

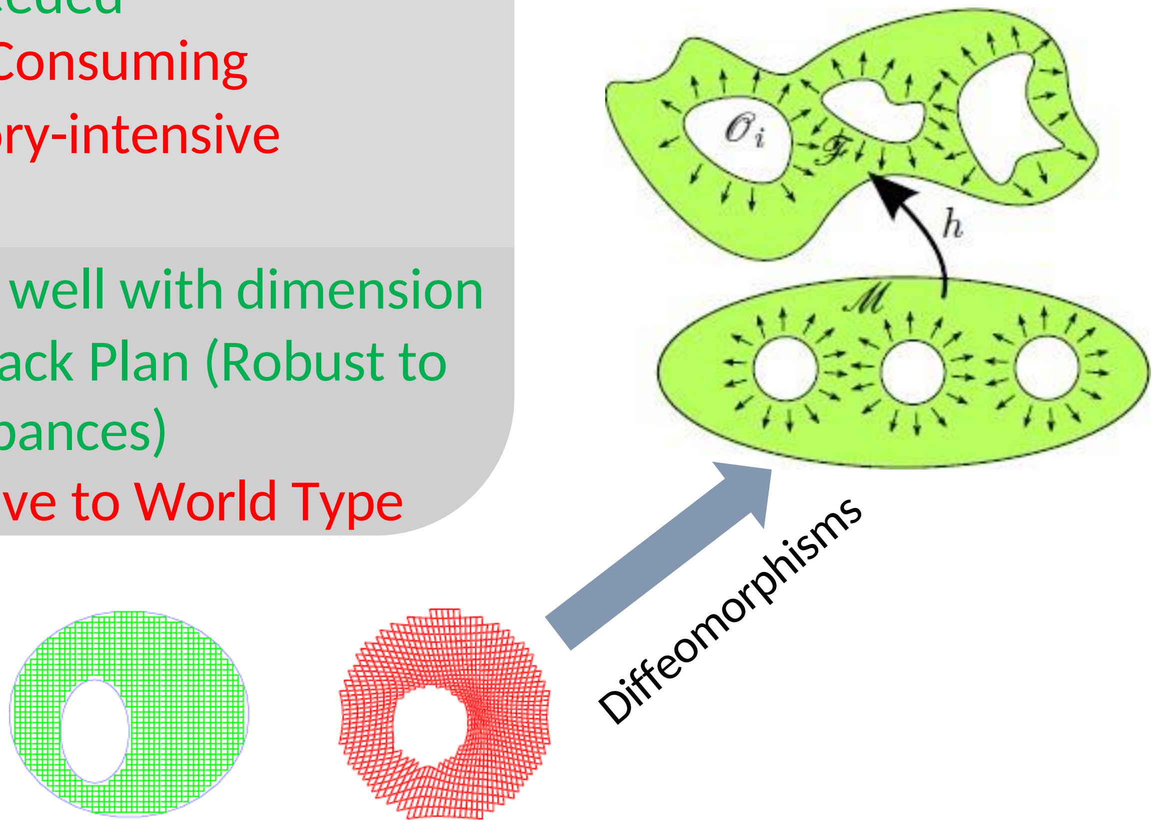
Characteristics

Combinatorial:
- Representative of discretization
- Curse of Dimensionality

Sampling-Based:
- Obstacle representation not needed
- Time-Consuming
- Memory-intensive

Implicit:
- Scales well with dimension
- Feedback Plan (Robust to disturbances)
- Sensitive to World Type

Koditschek & Rimon 1990

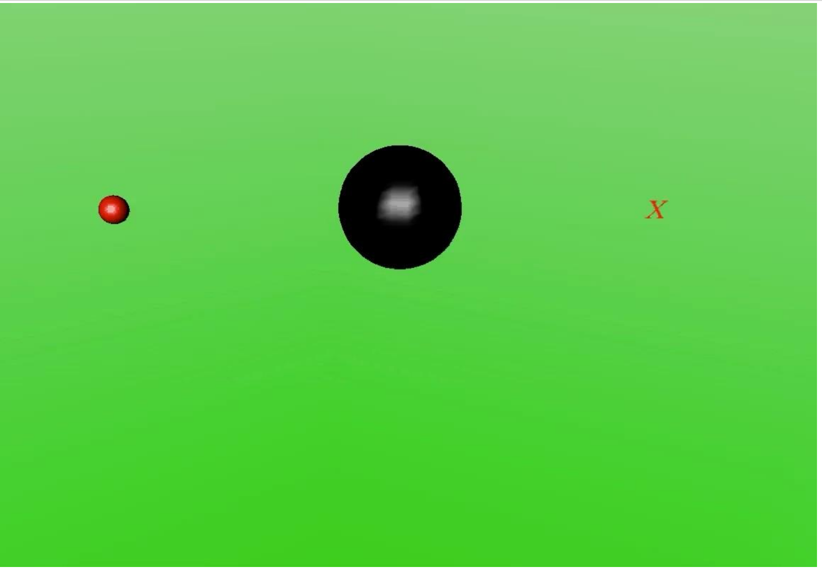


Navigation Function: Any C^2, admissible, polar, Morse function.

Existence: On any (analytic) Riemannian manifold with boundary.

Construction: Difficult. Proposed one for sphere worlds.

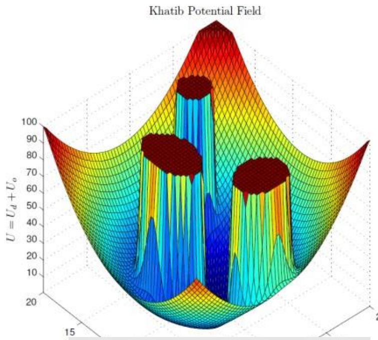
Single Agent: Holonomic Motion Control in Cluttered Environments



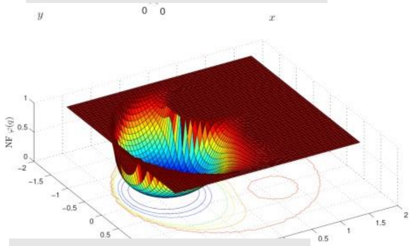
Potential Fields
(Khatib 1986)

↓

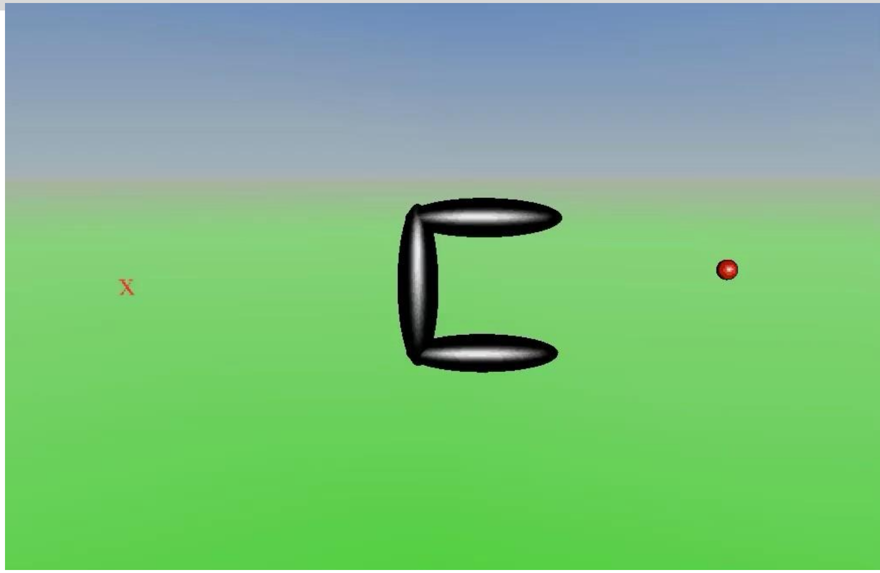
Navigation Functions
(Koditschek-Rimon 1990)



Local Minima

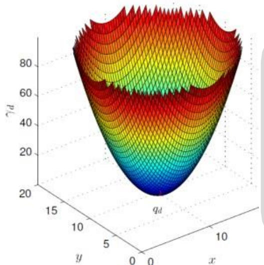


~~Local Minima~~



A **Navigation Function** (NF) $\varphi(q)$ is a special kind of potential function with a unique minimum q_G, the desired final configuration of a robot.

$$\varphi \triangleq \frac{\gamma_d}{\sqrt[k]{\gamma_d^k + \beta}}$$



$\gamma_d(q, q_d)$

Destination Function: Attracts

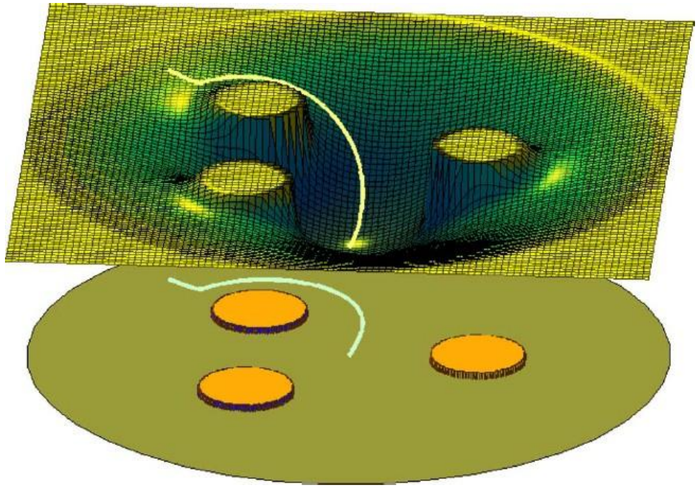
$\beta(q)$

Obstacle Function: Repels

Control Law:

$$\dot{q}(t) = u(t)$$

$$u(t) = -\nabla_q \varphi(q(t))$$



$$q \xrightarrow[t \to \infty]{} q_G$$

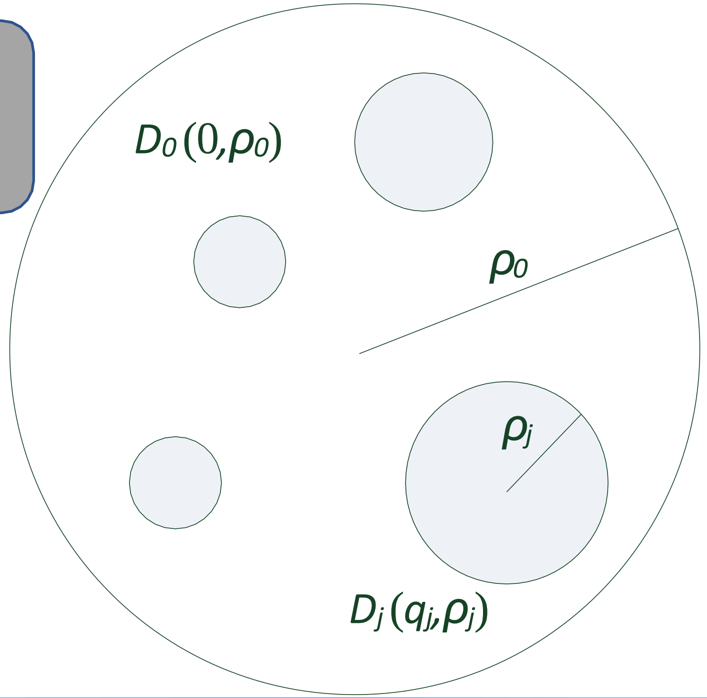
Single Agent: Holonomic Motion Control in Cluttered Environments

- There exists a lower bound N, so that for $\kappa \geq N$,

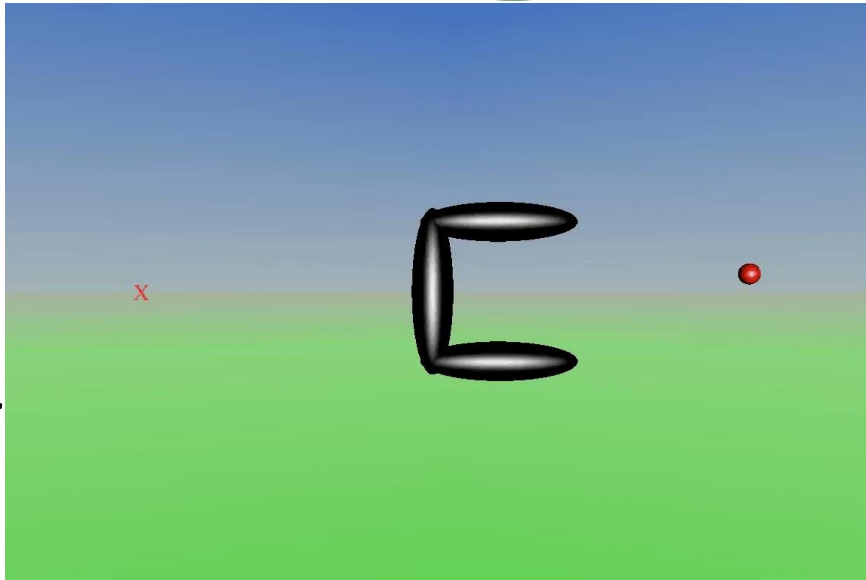
Attraction: to the Goal

Repulsion: from the Obstacles

$$\varphi(q) = \left(\frac{\|q - q_G\|^{2\kappa}}{\|q - q_G\|^{2\kappa} + \prod_{i=0}^{M} \beta_i(q)} \right)^{\frac{1}{\kappa}}$$



- is an NF i.e. it satisfies the requirements for global convergence with simultaneous collision avoidance, being
  - Analytic
  - Polar (single minimum)
  - Admissible (uniform max value at the "borders")
  - Morse (non singular 2nd derivative at critical points)
- Convergence is guaranteed from "almost every" initial point, as each of the M obstacles introduces a saddle point to the NF
- Spherical representation is the easiest to handle, however using ***conformal mapping*** "asteroid worlds" can be handled.



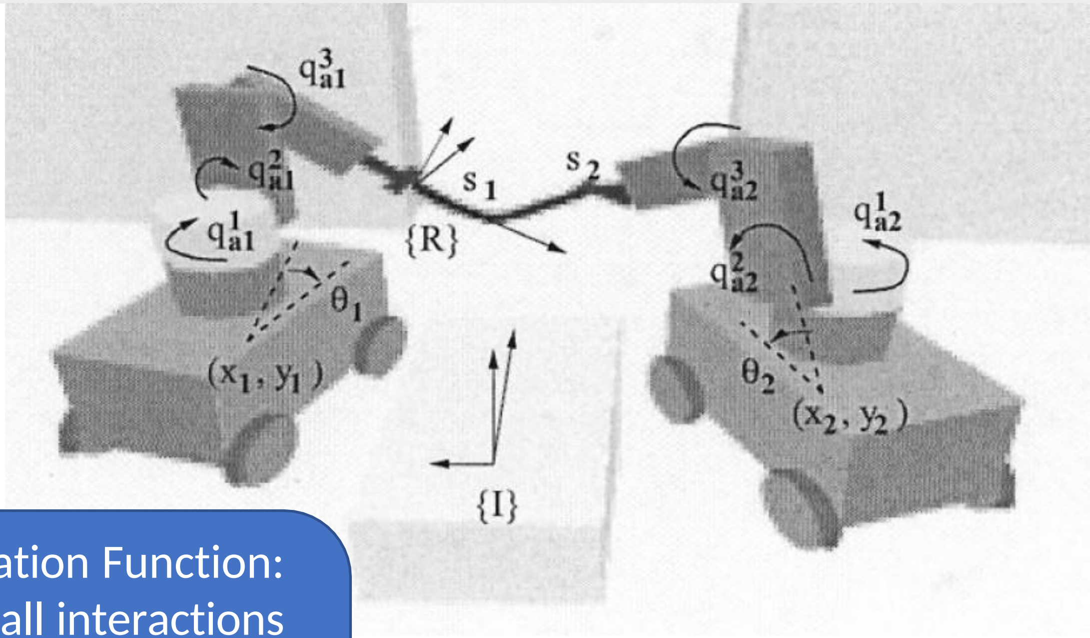
Motion Planning: The Single Agent case

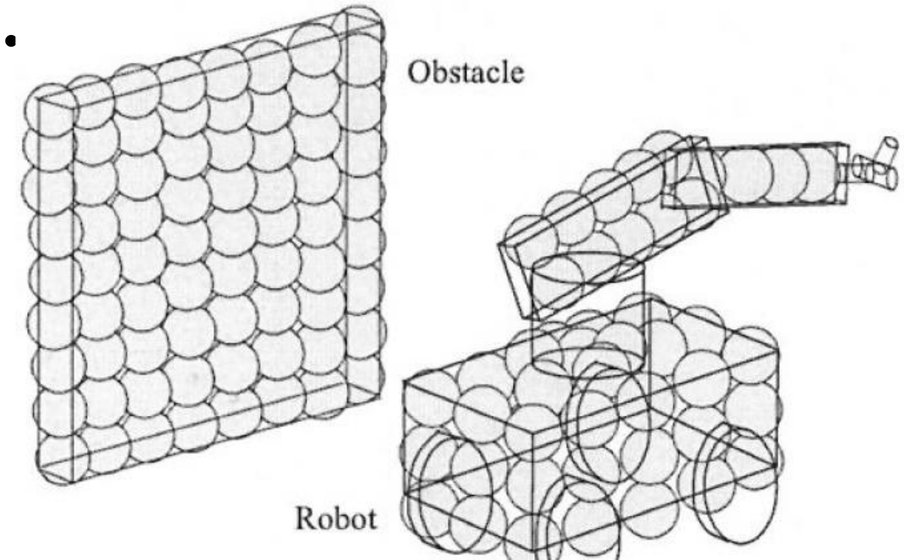
Kinematics / Environment	HOLONOMIC	Non-HOLONOMIC
Free	D. Kodistchek & E. Rimon	
Cluttered		

Single Agent: Nonholonomic Motion Control in Cluttered Environments

- The first motion planning methodology applicable to
  - articulated,
  - nonpoint
  - nonholonomic
- robots with guaranteed
  - collision avoidance and
  - convergence properties.
- Based on
  - a new class of nonsmooth Lyapunov functions and
  - a novel extension of the navigation function method
-

$$\dot{x}_r = v_r \cos\theta_r$$
$$\dot{y}_r = v_r \sin\theta_r$$
$$\dot{\theta}_r = \omega_r$$
$$\dot{\boldsymbol{q}}_{a_r} = \boldsymbol{u}_{a_r}, \qquad r = 1, \ldots, k$$





Appropriate Navigation Function:

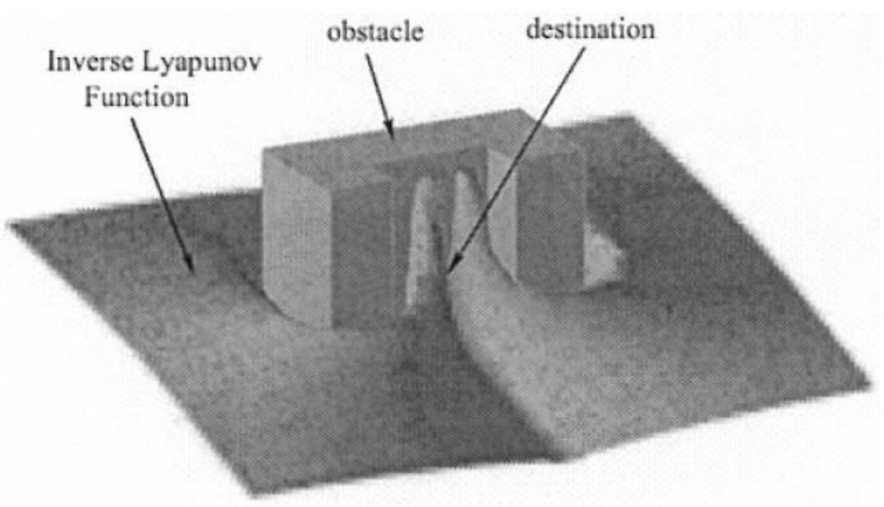
- It encapsulates all interactions between the robots, and
- appropriately penalizes the distances from the desired configuration and the obstacles

$$\varphi = \frac{\prod_p \|\boldsymbol{h}_2^p\|^2}{\left[\prod_p \|\boldsymbol{h}_2^p\|^{2k_v} + \prod_p d_p(\boldsymbol{h}_2^p)\right]^{1/k_v}}$$

H.Tanner, S.Loizou and K.J. Kyriakopoulos "Nonholonomic Navigation and Control of Cooperating Mobile Manipulators", *IEEE Trans. On Robotics & Automation*, Feb 2003, Volume: 19, Issue: 1, page(s): 53- 64

Single Agent: Nonholonomic Motion Control in Cluttered Environments

- The introduced Dipolar Inverse Lyapunov Functions (**DILF**) are appropriate for nonholonomic control and offer superior performance characteristics compared to existing tools. DILF features:
  - Non-smooth
  - Positive Semidefinite,
  - Vanishing on the boundary of the admissible space,
  - Tending to infinity at q_G ,
  - and designed so that the potential filed at the origin is aligned to the direction of the desired orientation
- The new potential field technique
  - uses diffeomorphic transformations and
  - exploits the resulting point-world topology.



DILF

Control Law

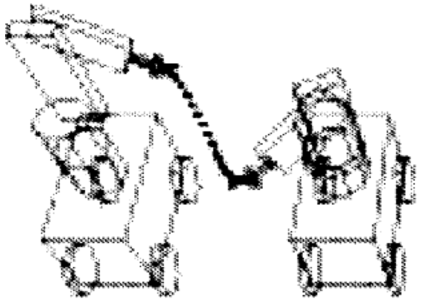
$$\theta_{d_r} \triangleq \operatorname{atan2}\left(-\operatorname{sign}(x_r) f_{y_r},\ -\operatorname{sign}(x_r) f_{x_r}\right)$$

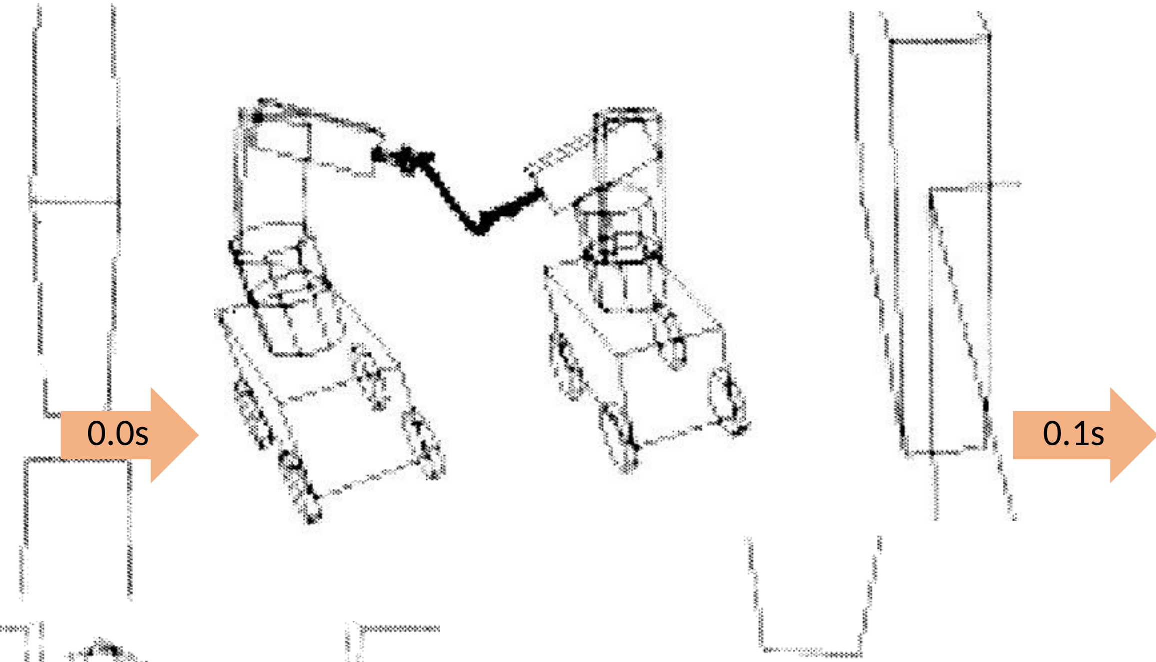
$$w_r \triangleq v_r \left(f_{x_r} \cos\theta_r + f_{y_r} \sin\theta_r\right) + k_o f_{\theta_r} \left(\theta_{d_r} - \theta_r\right)$$

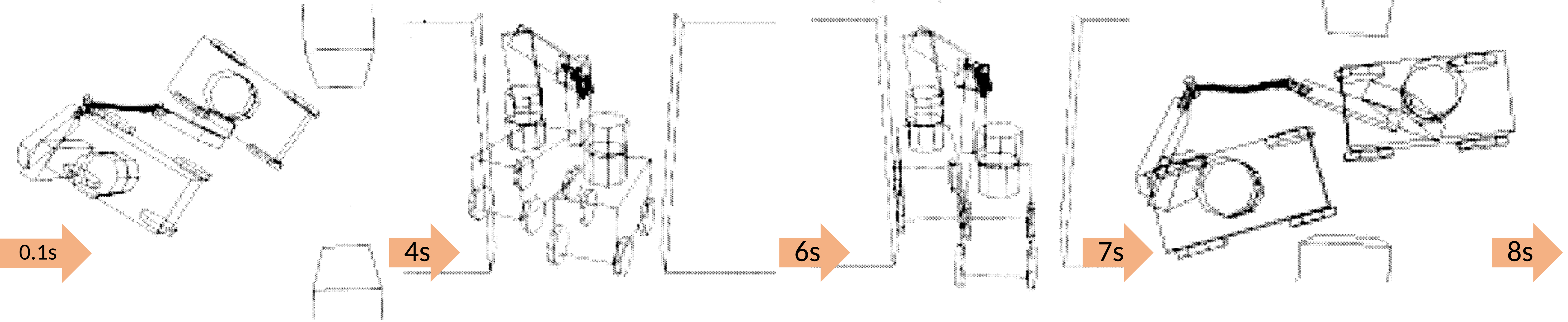
H.Tanner, S.Loizou and K.J. Kyriakopoulos "Nonholonomic Navigation and Control of Cooperating Mobile Manipulators", *IEEE Trans. On Robotics & Automation*, Feb 2003, Volume: 19, Issue: 1, page(s): 53- 64

Single Agent: Nonholonomic Motion Control in Cluttered Environments

- The combined approach is applied to the problem of
  - handling deformable material
  - by multiple nonholonomic mobile manipulators
  - in an obstacle environment
  - to yield a centralized coordinating control law.

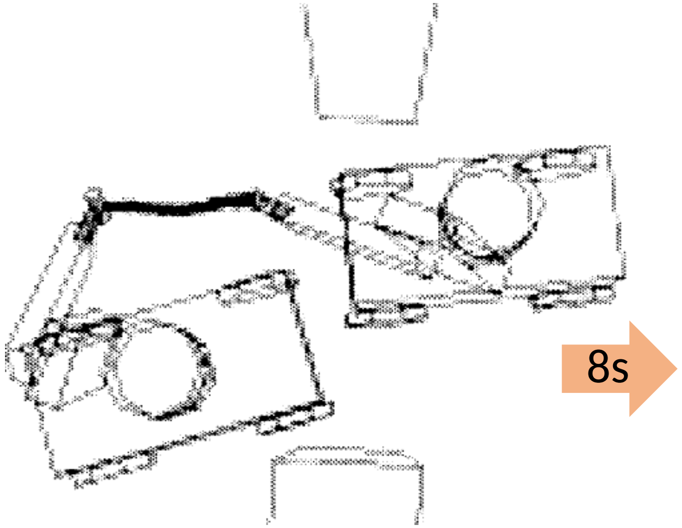


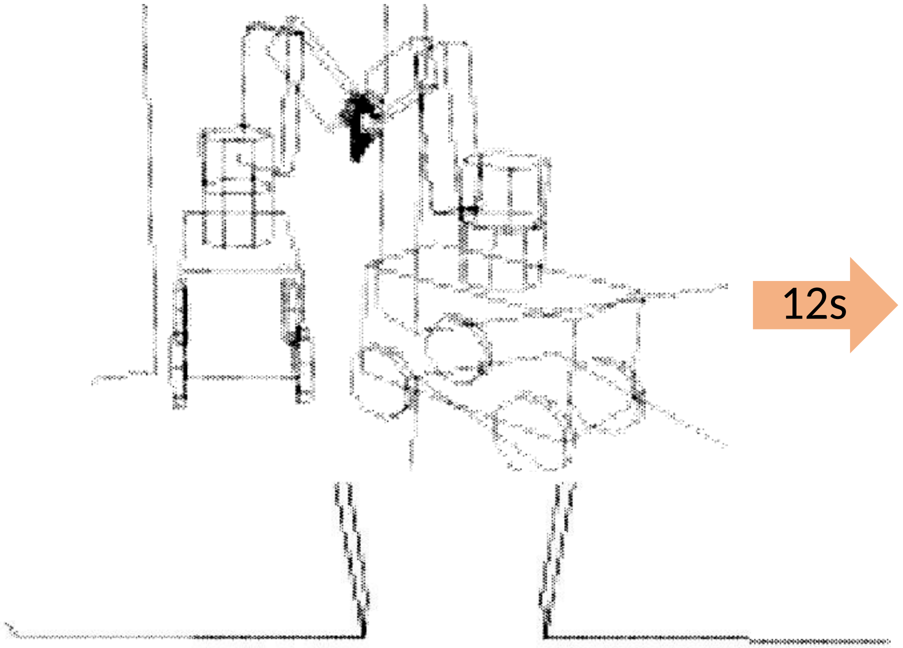


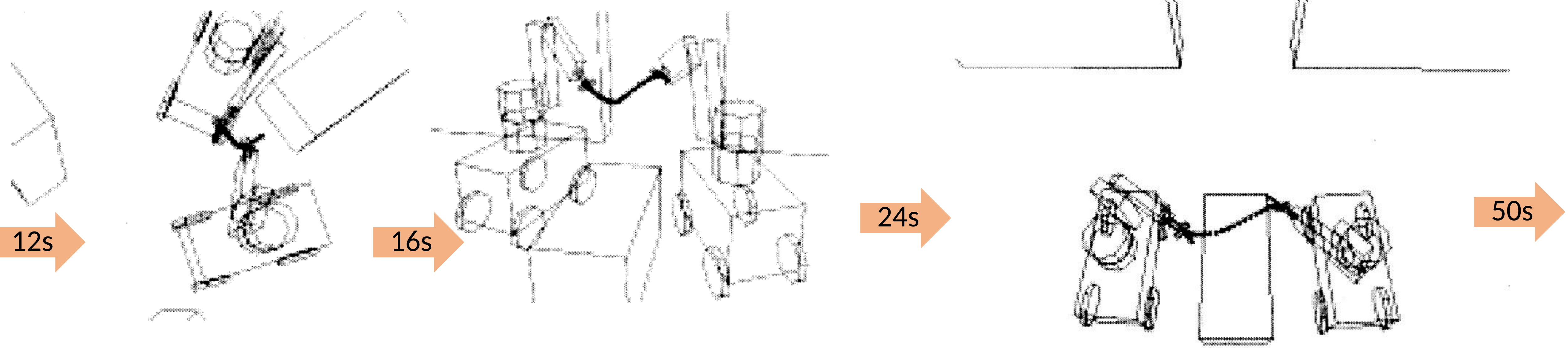


Single Agent: Nonholonomic Motion Controlin Cluttered Environments

- Simulation results verify
  - asymptotic convergence of the robots,
  - obstacle avoidance,
  - boundedness of object deformations, and
  - singularity avoidance for the manipulators.







Motion Planning: The Single Agent case

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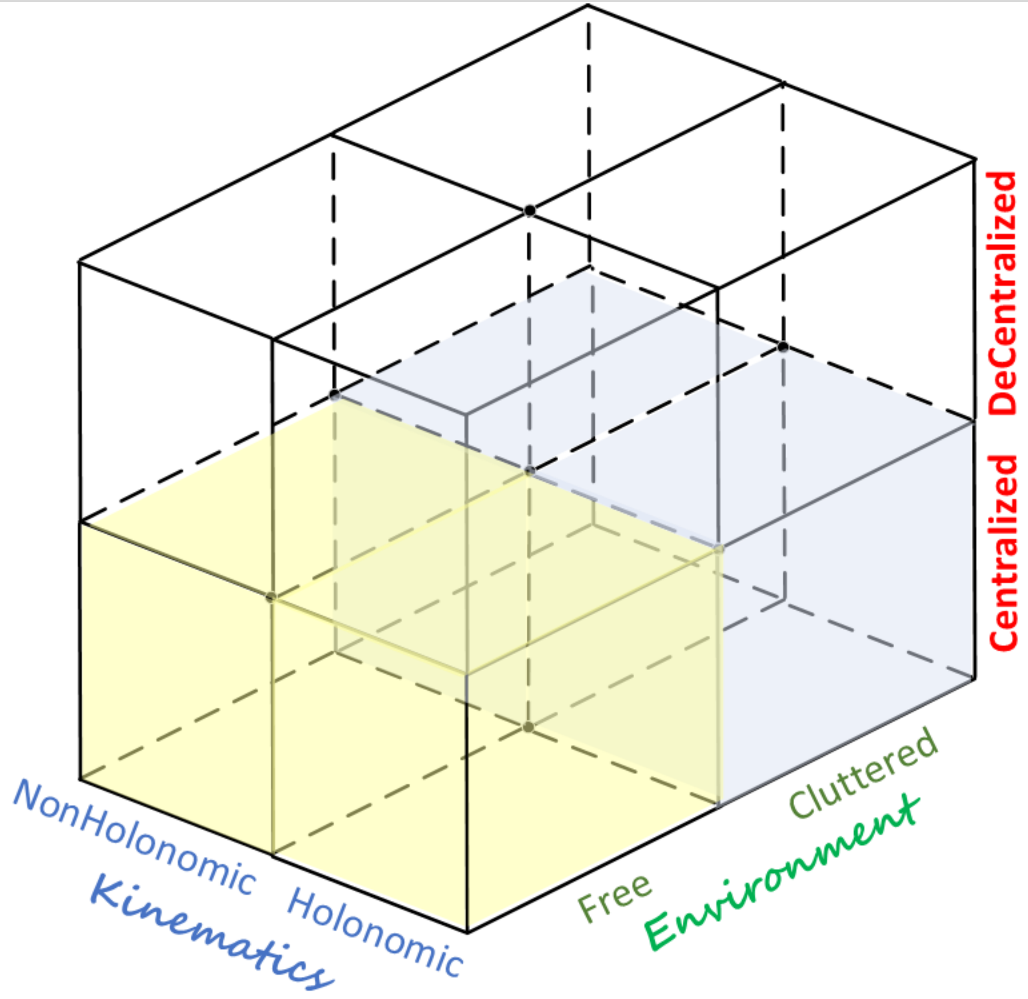
Motion Planning: The Multi - Agent case

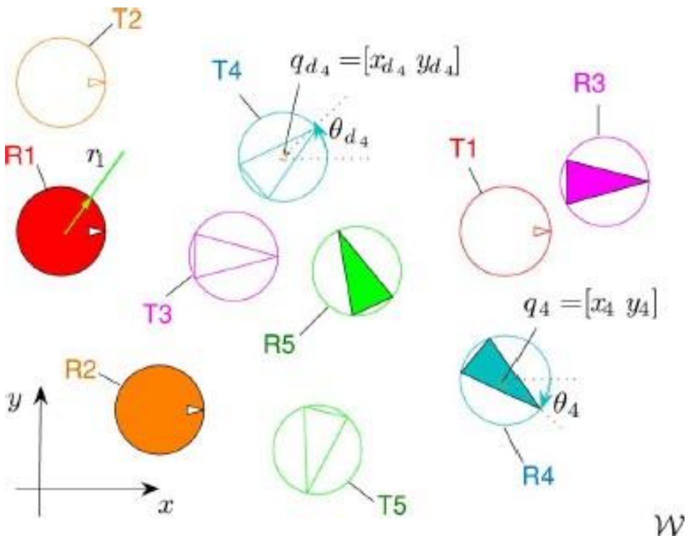


Multi Agent: Centralized control for Holonomic Vehicles in Free Environments

- Features
  - centralized methodology
  - team of multiple robotic agents.
  - robot kinematics:
    - **Holonomic**
    - Non-Holonomic
- Approach: introduced
  - the unifying framework of "***Multirobot*** *Navigation Functions*"

$$\varphi(q)=\left(\frac{\left\|q-q_G\right\|^{2\kappa}}{\left\|q-q_G\right\|^{2\kappa}+\prod_{i=0}^{M}\beta_i(q)}\right)^{\frac{1}{\kappa}}$$

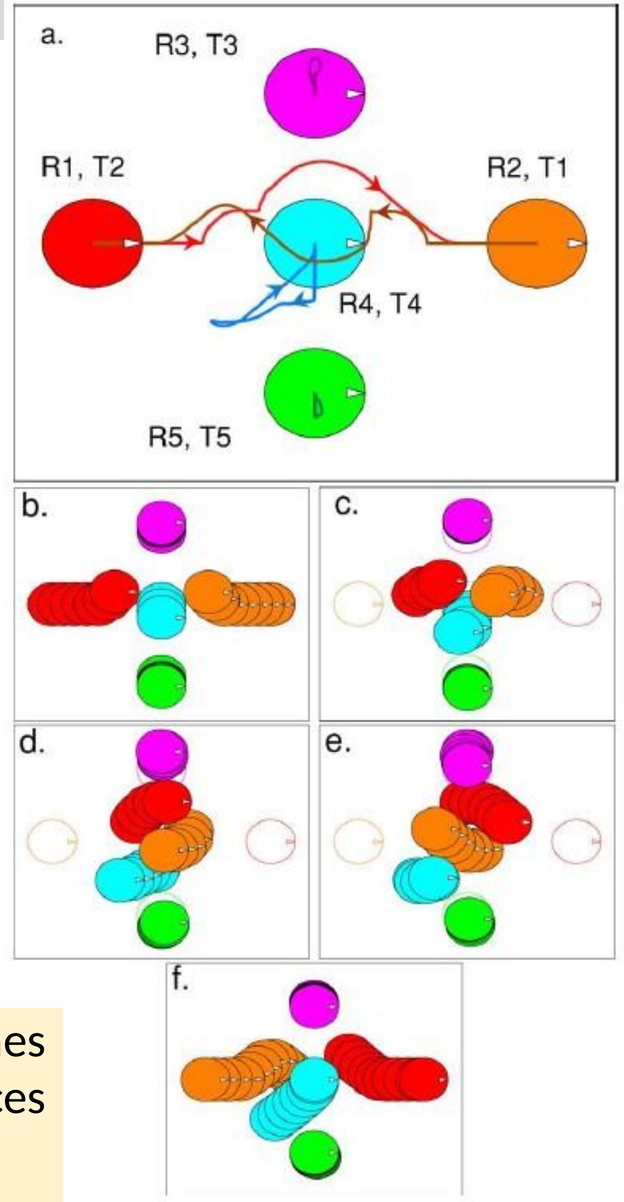
$$\varphi(q)=\frac{\left\|q-q_G\right\|^{2}}{\left(\left\|q-q_G\right\|^{2\kappa}+G\right)^{\frac{1}{\kappa}}}$$



Control of Holonomic Vehicles

$$\mathbf{u_h}=-K\cdot\nabla\varphi\left(\mathbf{q_h}\right)$$

G is to encapsulate the weight of all possible collision schemes between the members of a team by considering the distances between robots, expressed in a single scalar function



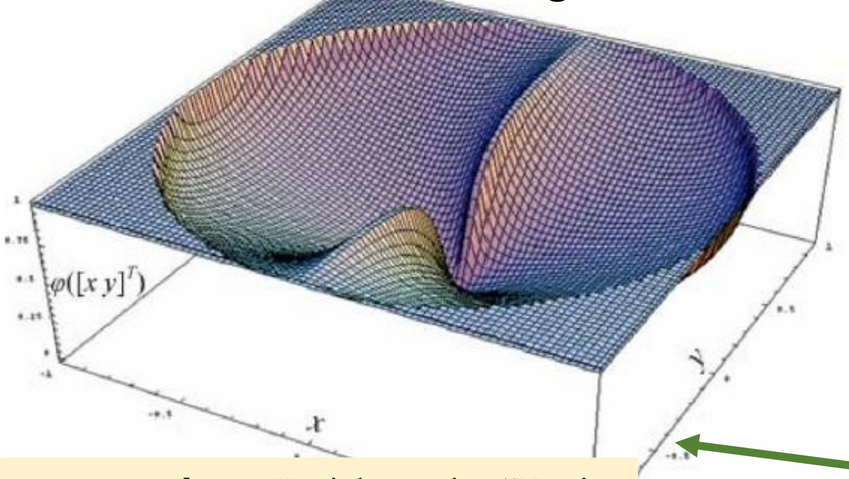
Multi Agent: Centralized control for Non-Holonomic Vehicles in Free Environments

- Features
  - centralized methodology
  - team of multiple robotic agents.
  - robot kinematics:
    - Holonomic
    - **Non-Holonomic**

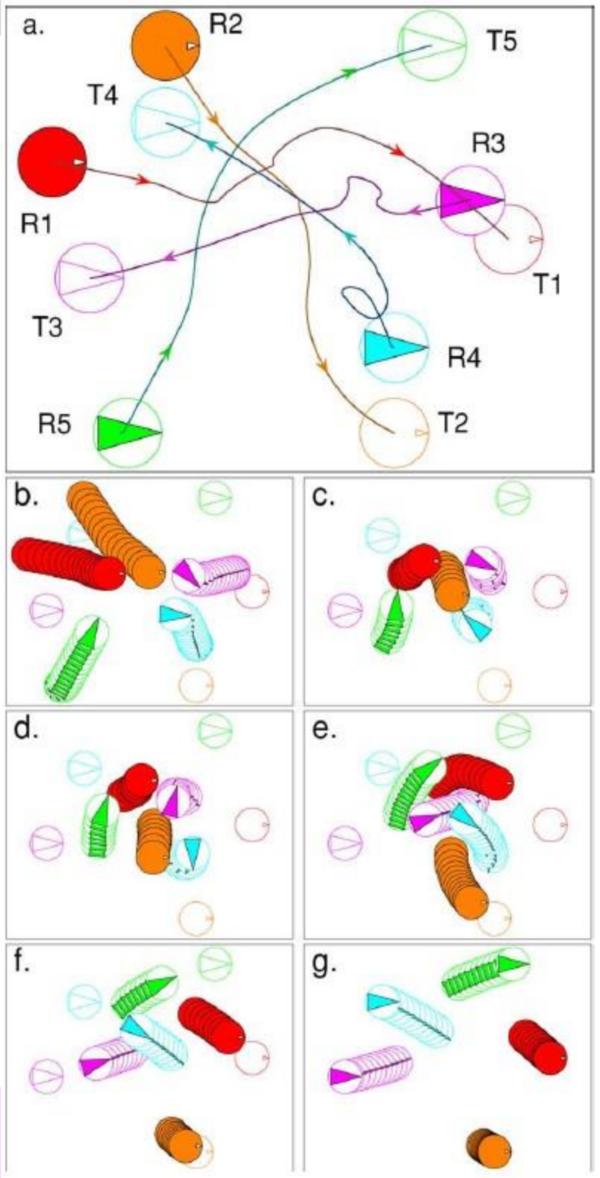
$$\varphi(q) = \frac{\left\|q - q_G\right\|^2}{\left(\left\|q - q_G\right\|^{2\kappa} + G\right)^{\frac{1}{\kappa}}}$$

- Approach: through
  - the ”***Multirobot*** *Navigation Functions*” framework

$$\varphi(q) = \frac{\left\|q - q_G\right\|^2}{\left(\left\|q - q_G\right\|^{2\kappa} + H_{nh} \cdot G\right)^{\frac{1}{\kappa}}}$$



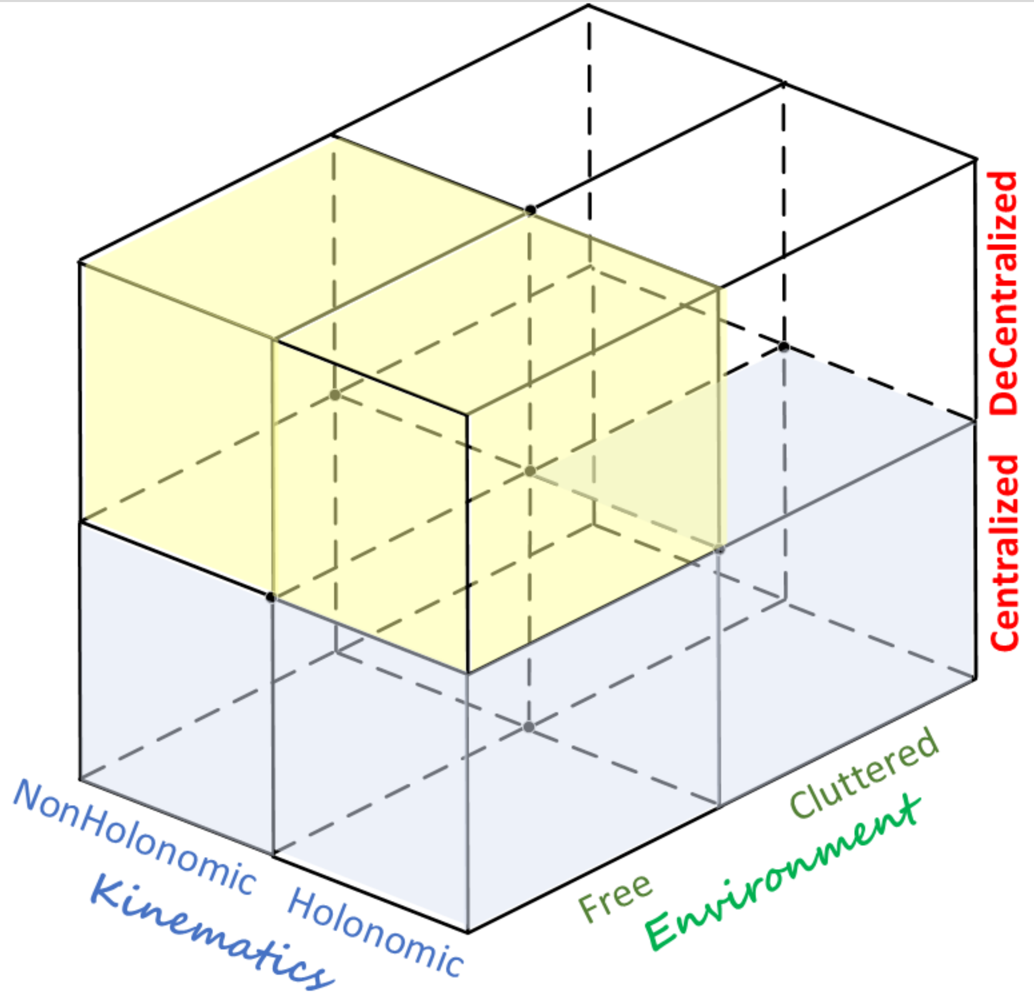
Formed as a “pseudo-obstacle” leading to the ***Dipolar Navigation Function***



S.G. Loizou and K.J. Kyriakopoulos “Navigation of Multiple Kinematically Constrained Robots”, *IEEE Trans on Robotics*, vol. 24 (1), pp. 221-31, Feb. 2008

Leads to a controller which is globally asymptotically stable a.e. i.e. everywhere except for a set of initial conditions of measure zero that lead to saddle points.

Motion Planning: The Multi - Agent case



MultiAgent: towards Decentralized control in Free Environments

- Decentralization: concerns the knowledge of each agent for the rest of the team regarding their
  - State,
  - Objectives, and
  - Actions
- Features
  - Each agent is not aware of the *desired destination* of the others
  - *Limited sensing* capabilities of each agent: each agent has only partial knowledge of the state space.
  - *Kinematics*: holonomic / nonholonomic
  - Extension to agent *dynamic model* approximation.
- Advantages of the proposed scheme:
  - Relatively *low complexity* wrt the number of agents (compared to centralized approaches), and
  - Applicability to *non-point* agents.

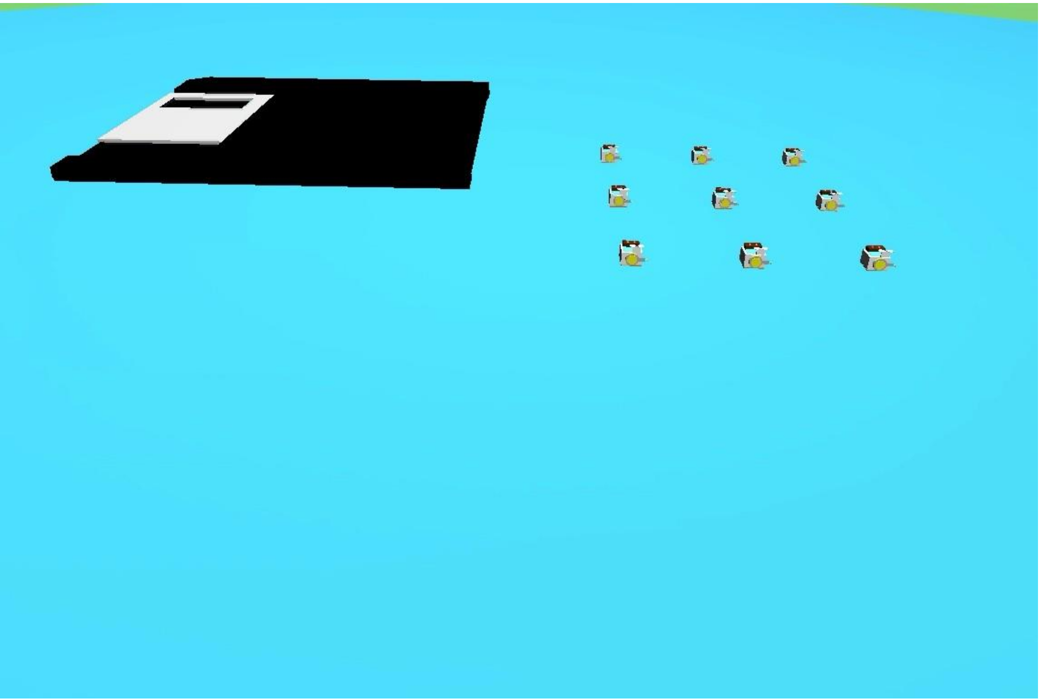
Multi Agent: towards Decentralized control for Holonomic Vehicles in Free Environments

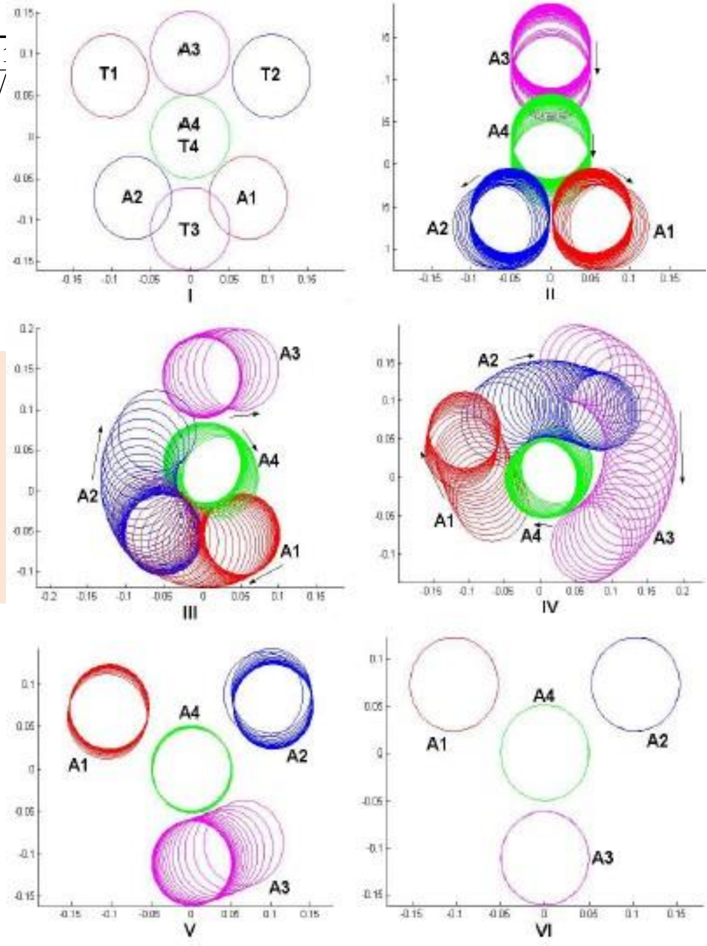
- Features
  - Each agent has knowledge only of its own desired destination but not of the others (decentralization)
  - Each agent has global knowledge of the position of the others at each time instant (local sensing, later)

$$\varphi_i(q) = \varphi_i(q_i, \tilde{q}_i) = \varphi_i(q_i, t) = \frac{\|q_i - q_{Gi}\|^2 + f_i}{\left(\left(\|q_i - q_{Gi}\|^2 + f_i\right)^k + G_i\right)^{1/k}}$$

Extension of the NF framework

To ensure "cooperation" i.e. φ_i attains positive values in proximity situations even when agent i has already reached its destination.





- Kinematics: holonomic
- *Spherical agents* – Workspace: bounded and spherical (no constraint due to analytic diffeomorphisms)

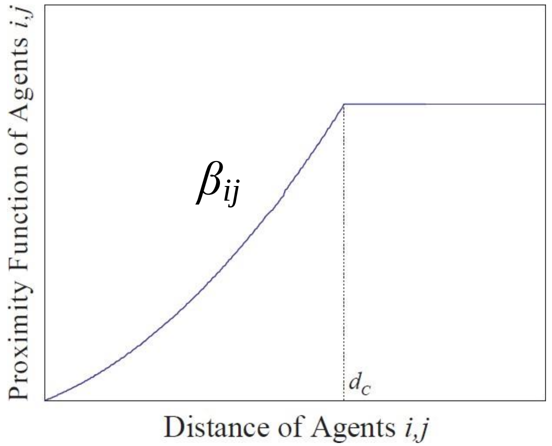
Control Law

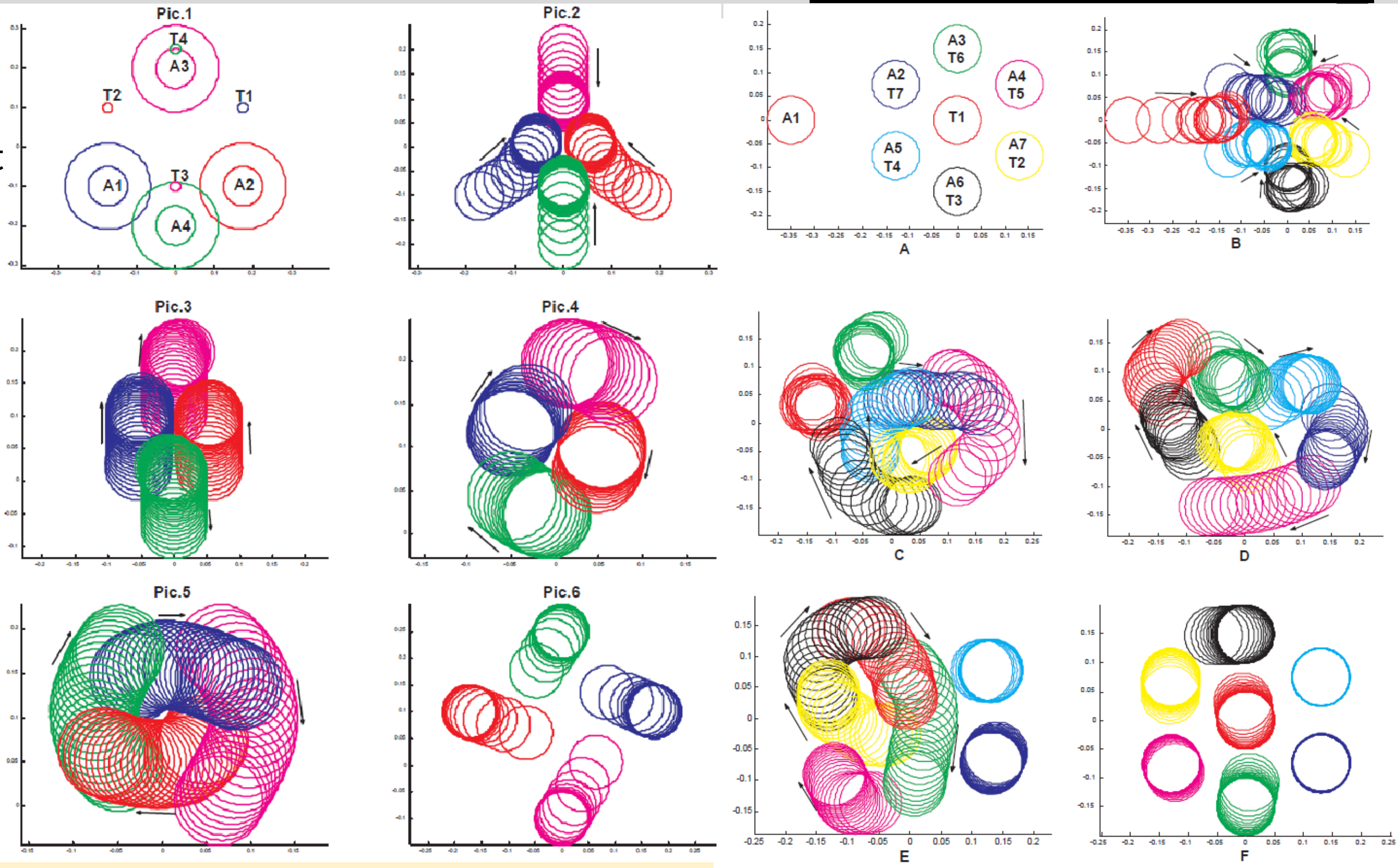
$$u_i = -K_i \cdot \nabla_i \varphi_i(q_i) \qquad K_i > 0$$

Multi Agent: towards Decentralized control for Holonomic Vehicles in Free Environments with limited sensing

- Features
  - Each agent has knowledge
    - only of its own desired destination but not of the others (decentralization)
    - local (i.e. within a sensing radius) of the position of the others at each time instant
    - the exact number N of agents in the workspace.
  - Kinematics: holonomic
  - Spherical agents – Workspace: bounded and spherical (no constraint due to analytic diffeomorphisms)

$$\varphi_i(q)=\varphi_i(q_i,\tilde{q}_i)=\varphi_i(q_i,t)=\frac{\|q_i-q_{Gi}\|^2+f_i}{\left(\left(\|q_i-q_{Gi}\|^2+f_i\right)^{\kappa}+G_i\right)^{\frac{1}{\kappa}}}$$





G_i encapsulates the weight of all possible collision schemes between the members of a team by considering the **proximity functions** between robots. It is such as to ensure that the gradient motion imposed on agent *i* under the **control law** is repulsive with respect to the boundary of the free space to guarantee collision avoidance

Control Law

$$u_i=-K_i\cdot\nabla_i\varphi_i(q_i)\qquad K_i>0$$

Multi Agent: towards Decentralized control for NonHolonomic Vehicles in Free Environments

- Features
  - Each agent has knowledge only of its own desired destination but not of the others
  - Each agent has global knowledge of the position of the others at each time instant
  - Kinematics: nonholonomic
  - Around the target of each agent there is a **safe region ε** only accessible by the agent, while regarded as an obstacle by the others.
  - *Spherical* agents – Workspace: bounded and spherical

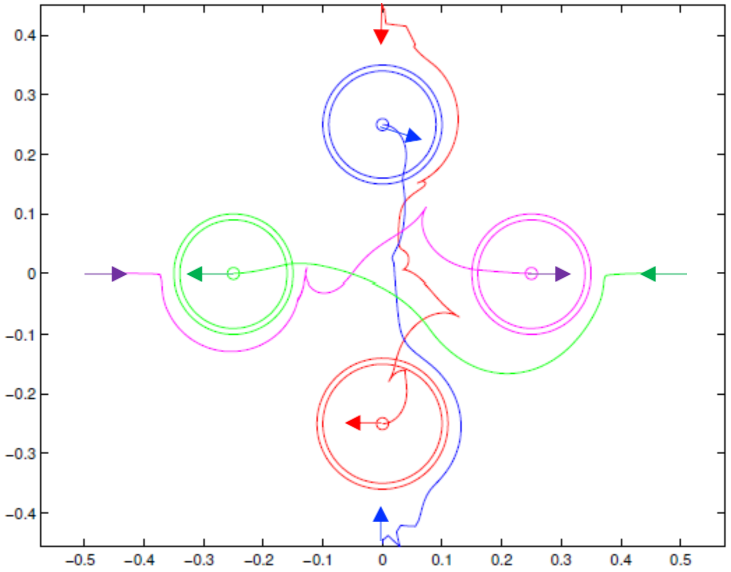
$$\dot{x}_i = u_i \cos\theta_i$$
$$\dot{y}_i = u_i \sin\theta_i$$
$$\dot{\theta}_i = \omega_i$$

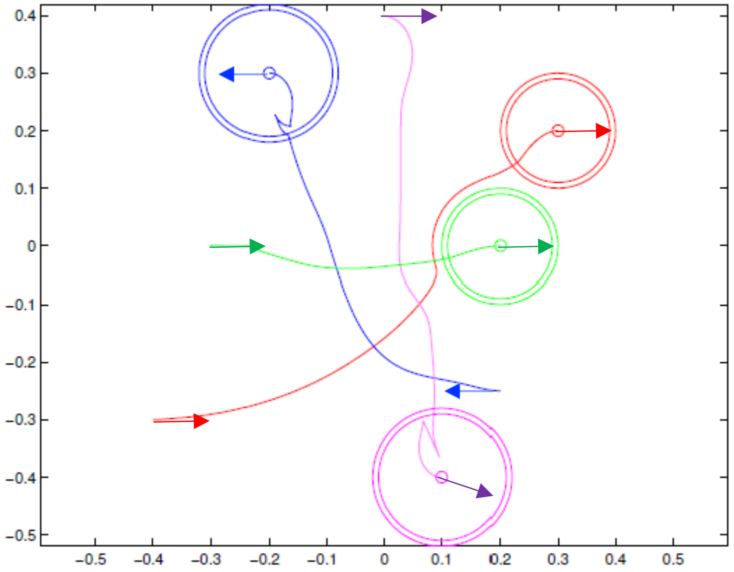
$$\varphi_i(q_i) = \frac{\left\|q_i - q_{Gi}\right\|^2}{\left(\left\|q_i - q_{Gi}\right\|^{2\kappa} + H_{nh_i} \cdot G_i \cdot b_{t_i}\right)^{\frac{1}{\kappa}}}$$

Formed as a “pseudo-obstacle” leading to the ***Dipolar Navigation Function***

$$b_{t_i} = \prod_{j \neq i} \left(\left\|q_i - q_{d_j}\right\|^2 - (\varepsilon + r_i)^2\right)$$

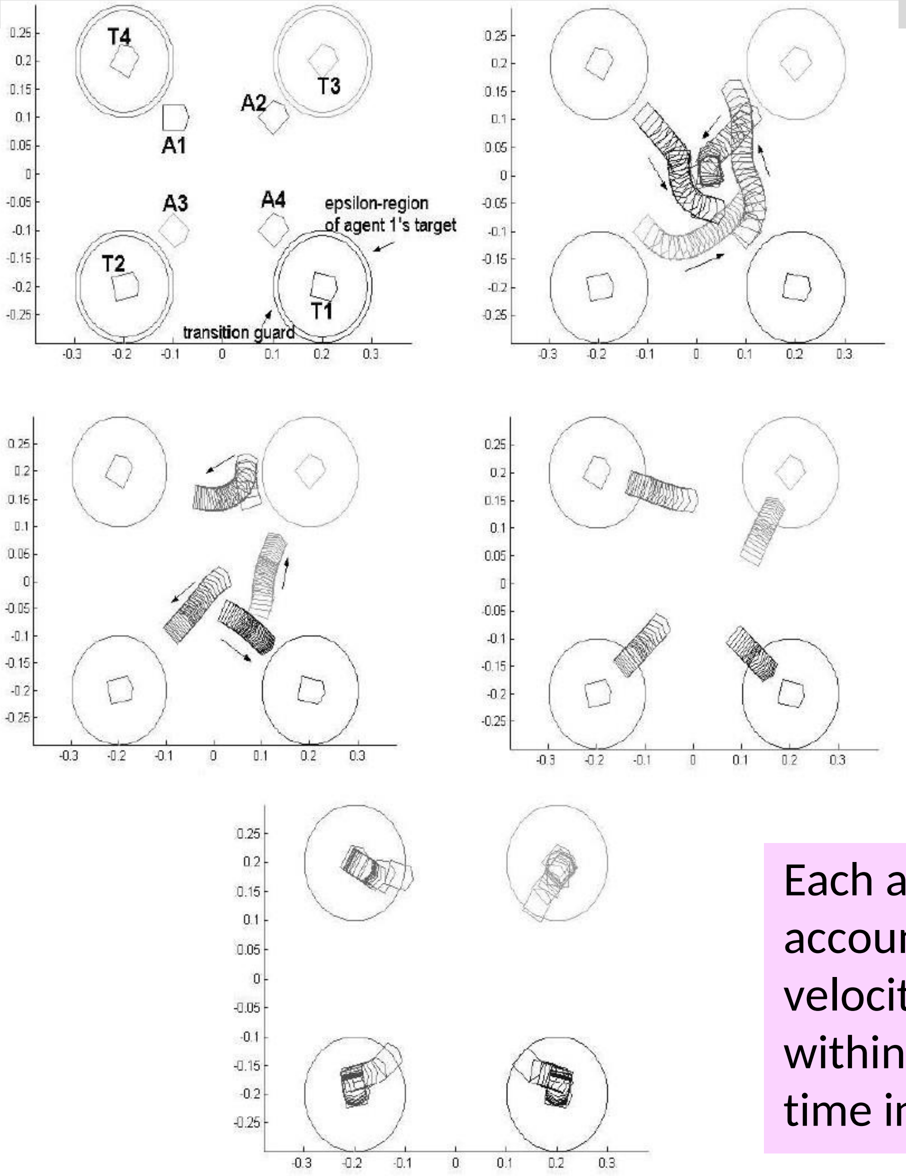
Safe Region





Multi Agent: towards Decentralized control for NonHolonomic Vehicles in Free Environments with limited sensing

- Features
  - Each agent has knowledge
    - only of its own desired destination but not of the others (decentralization)
    - local (i.e. within a sensing radius) of the position of the others at each time instant
    - the exact number N of agents in the workspace.
  - Kinematics: nonholonomic
  - Around the target of each agent there is a **safe region** ε only accessible by the agent, while regarded as an obstacle by the others.
  - Spherical agents – Workspace: bounded and spherical



Each agent has to take into account only the positions and velocities of agents that are within each sensing zone at each time instant

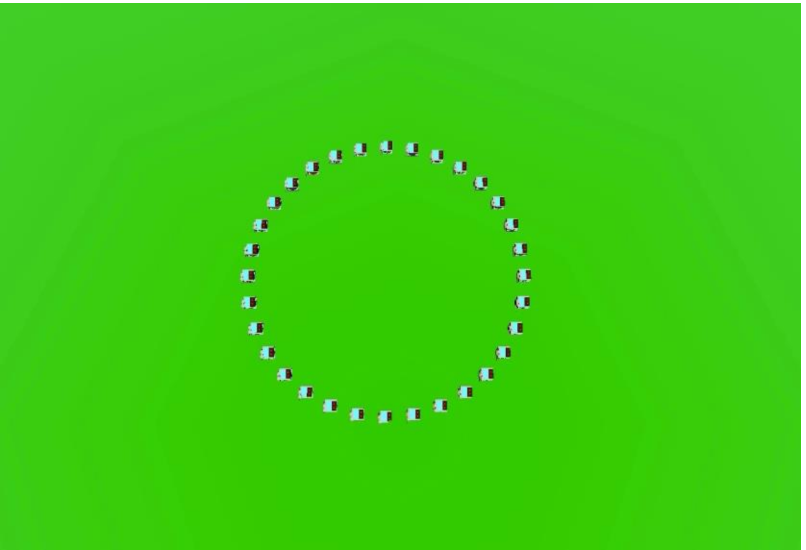
Multi Agent: towards Decentralized control for Dynamic Holonomic Vehicles in Free Environments

- Features
  - Each agent has knowledge only of its own desired destination but not of the others (decentralization)
  - Each agent has global knowledge of the position of the others at each time instant (local sensing, later)
  - Dynamic holonomic
  - *Spherical agents* – Workspace: bounded and spherical (no constraint due to analytic diffeomorphisms)

Double Integrator

$$\begin{aligned}\dot{q}_i &= v_i \\ \dot{v}_i &= u_i\end{aligned}, \; i \in \{1, \ldots, N\}$$

$$\varphi_i(q) = \varphi_i(q_i, \dot{q}_i) = \varphi_i(q_i, t) = \frac{\|q_i - q_{Gi}\|^2 + f_i}{\left(\left(\|q_i - q_{Gi}\|^2 + f_i\right)^{\kappa} + G_i\right)^{\frac{1}{\kappa}}}$$



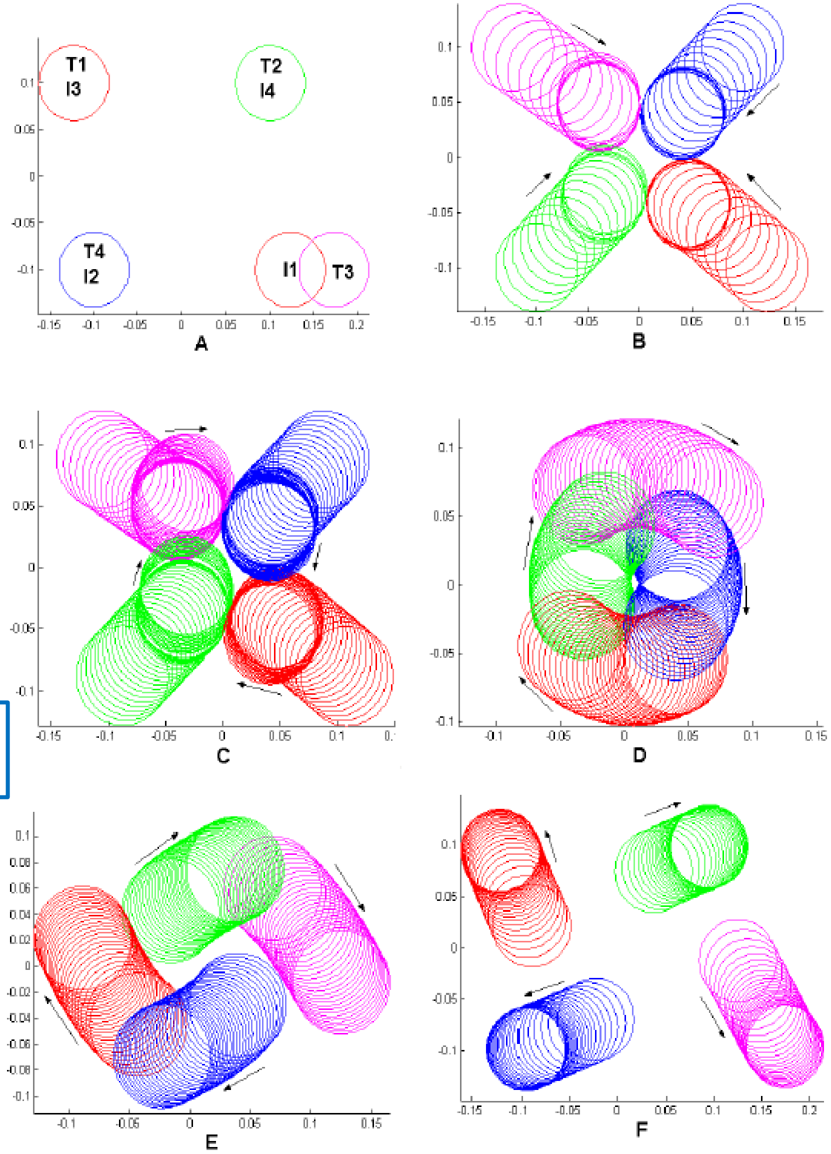
Control Law

$$u_i = -K_i \frac{\partial \varphi_i}{\partial q_i} + \theta_i\left(v_i, \frac{\partial \varphi_i}{\partial t}\right) - g_i v_i$$

$$\theta_i\left(v_i, \frac{\partial \varphi_i}{\partial t}\right) \triangleq -\frac{c v_i}{\tanh\left(\|v_i\|^2\right)} \left|\frac{\partial \varphi_i}{\partial t}\right|$$

$$K_i, g_i > 0$$

$$\frac{\partial \varphi_i}{\partial t} = \sum_{j \neq i} \frac{\partial \varphi_i}{\partial q_j} \dot{q}_j$$



Multi Agent: towards Decentralized control for Dynamic NonHolonomic Vehicles in Free Environments

- Features
  - ➢ Each agent has knowledge only of its own desired destination but not of the others
  - ➢ Each agent has global knowledge of the position of the others at each time instant
  - ➢ Kinematics: Dynamic nonholonomic
  - ➢ Spherical agents – Workspace: bounded and spherical

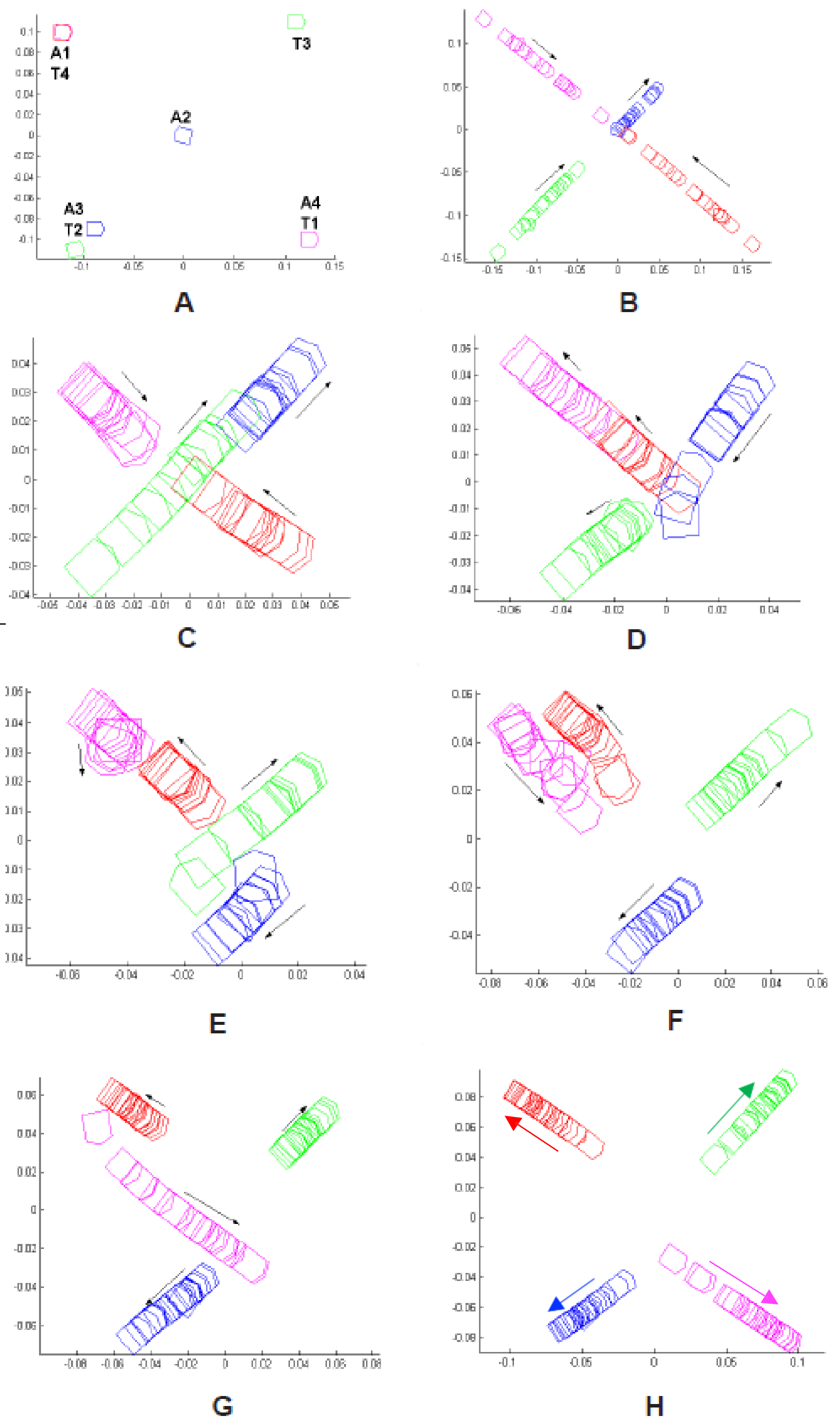
$$\dot{x}_i = v_i \cos\theta_i$$
$$\dot{y}_i = v_i \sin\theta_i$$
$$\dot{\theta}_i = \omega_i \qquad , i \in \{1, \dots, N\}$$
$$\dot{v}_i = u_i$$

$$\varphi_i(q_i) = \frac{\|q_i - q_{Gi}\|^2 + f_i}{\left[\left(\|q_i - q_{Gi}\|^2 + f_i\right)^{\kappa} + H_{nh_i} \cdot G_i\right]^{\frac{1}{\kappa}}}$$

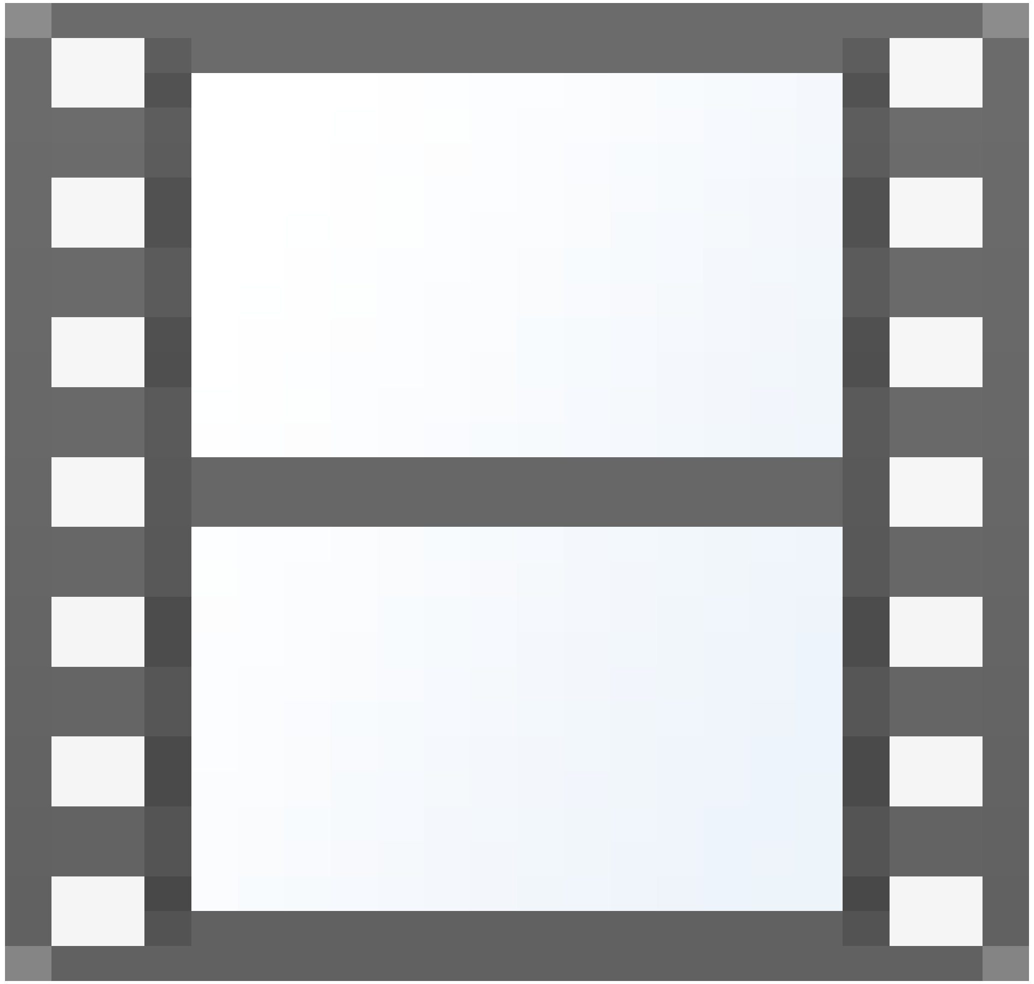
Formed as a "pseudo-obstacle" leading to the ***Dipolar Navigation Function***

Control Law (asymptotically stable)

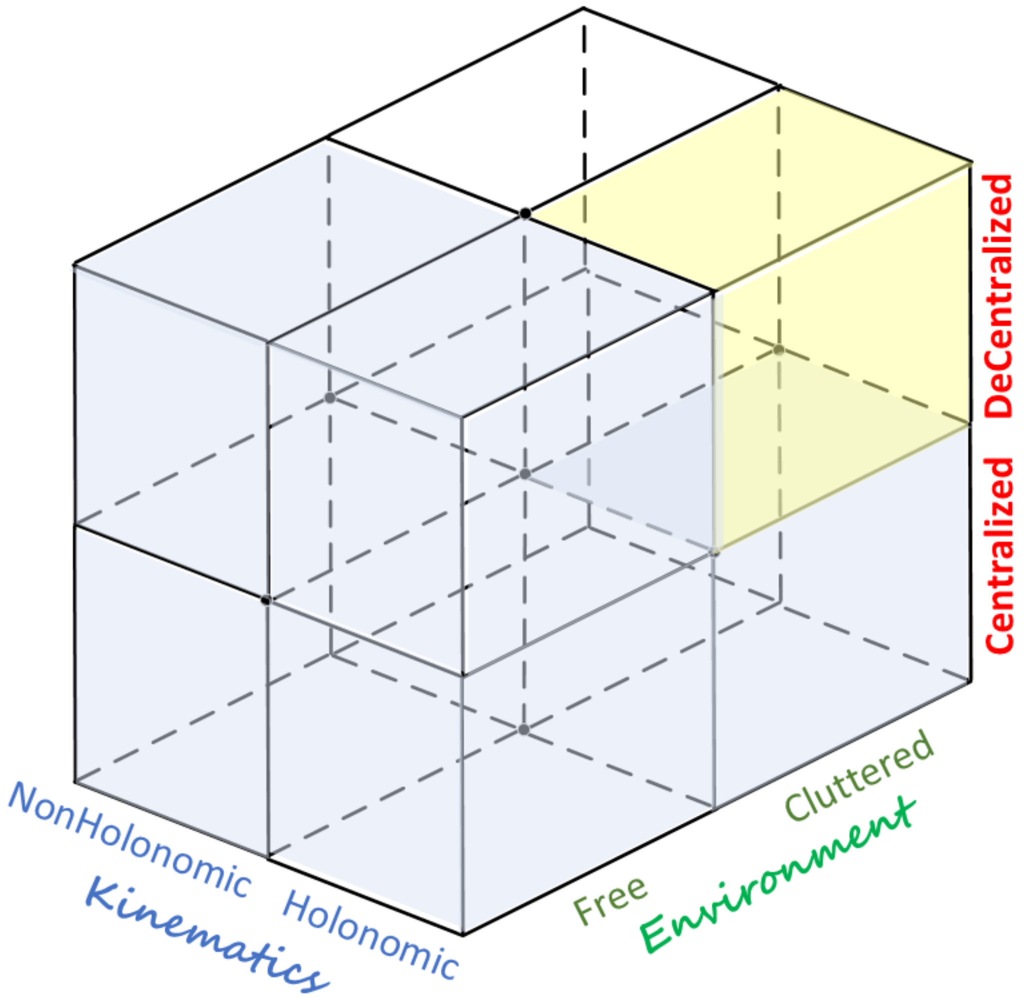
$$u_i = -v_i\{|\nabla_i \varphi_i \cdot \eta_i| + M_i\} - g_i v_i - \frac{v_i}{\tanh(|v_i|)} K_{v_i} K_{z_i}$$
$$\omega_i = -K_{\theta_i}(\theta_i - \theta_{di} - \theta_{nh_i}) + \dot{\theta}_{nh_i}$$



MultiAgent: towards Decentralized control for Dynamic NonHolonomic Vehicles in Free Environments

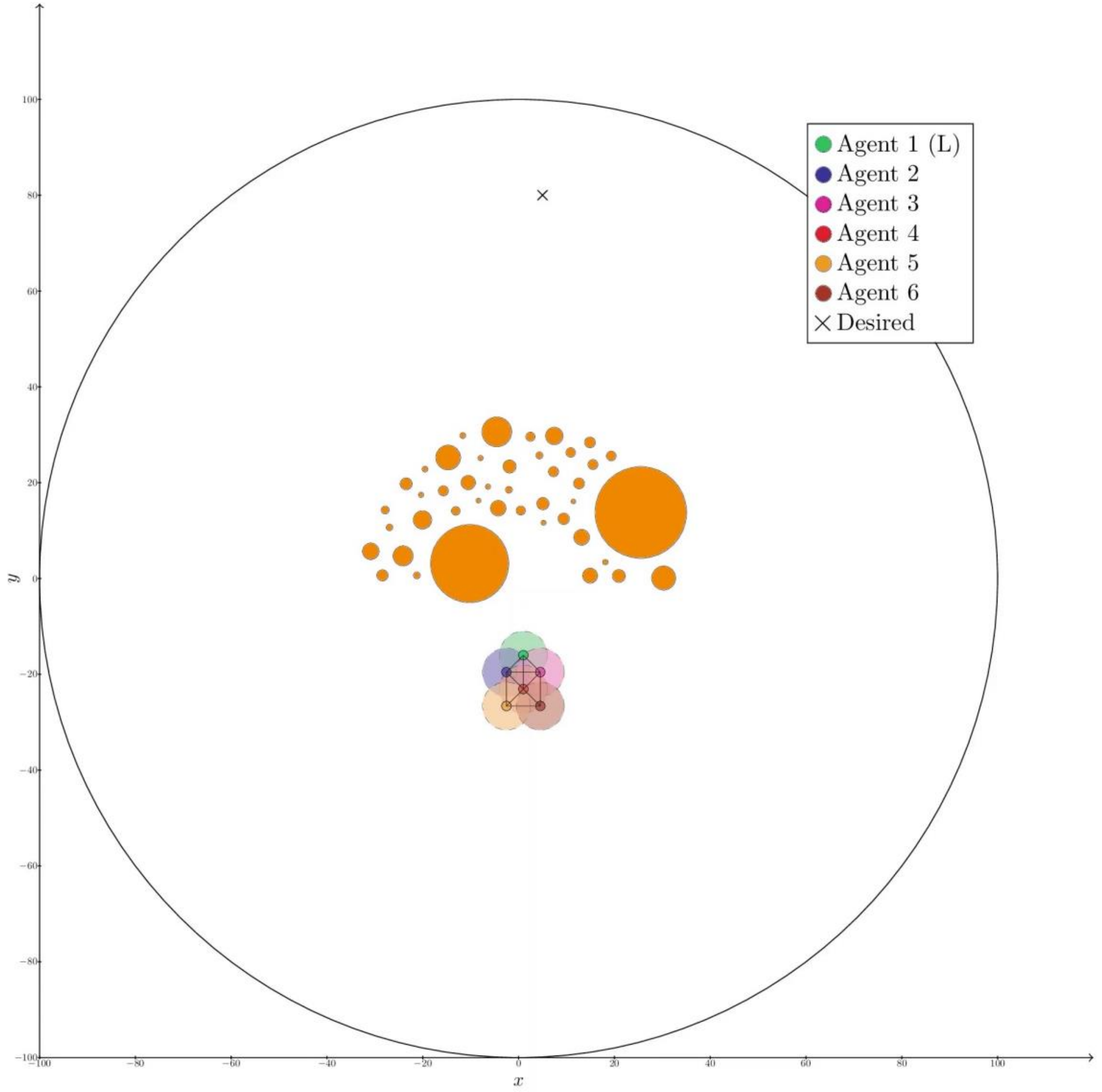


Motion Planning: The Multi - Agent case



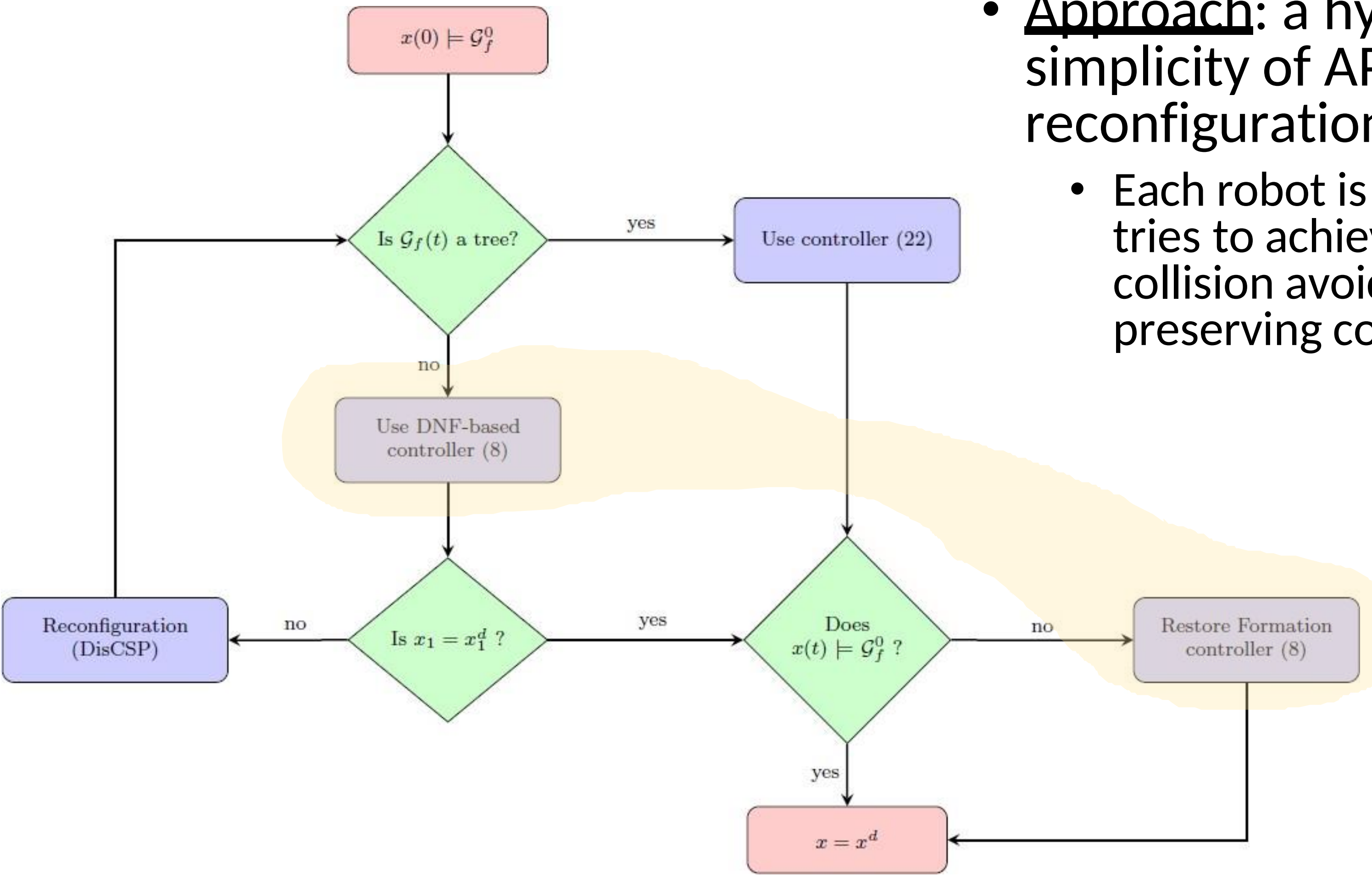
Multi Agent: Decentralized control of Holonomic Vehicles in Cluttred Environments

- <u>Problem</u>: formation control for a team of
  - non-point robots with local
    - ✓ sensing and
    - ✓ communication
  - in a "*single leader-multiple followers*" architecture,
  - where the leader is tasked with navigating the team to a predefined location
  - in a cluttered workspace.
- <u>Features</u>: We propose a distributed *reconfiguration strategy* of the set of connectivity and formation specifications that
  - assures convergence to the desired point,
  - while guaranteeing global connectivity.
- <u>Approach</u>: a hybrid control scheme, combining the simplicity of APF methods with a discrete reconfiguration algorithm.

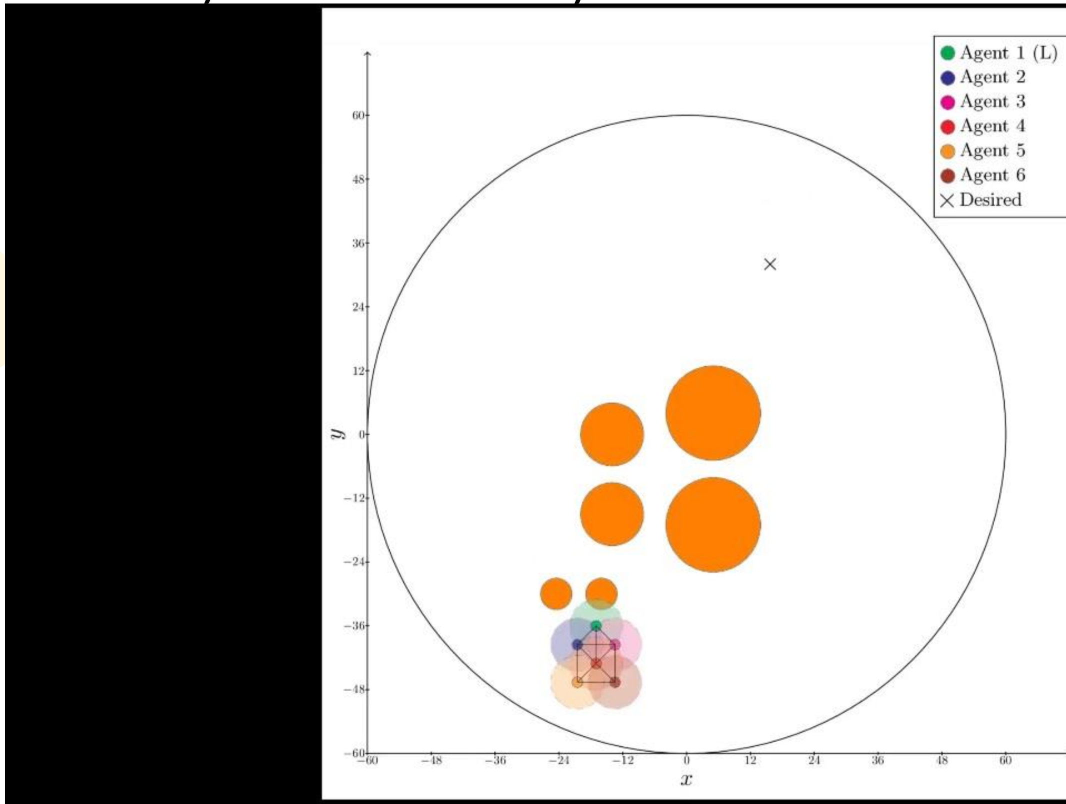


MultiAgent: Decentralized control of Holonomic Vehicles in Cluttered Environments

- Approach: a hybrid control scheme, combining the simplicity of APF methods with a discrete reconfiguration algorithm:
  - Each robot is equipped with an APF based controller which tries to achieve the desired local formation while ensuring collision avoidance with other agents and static obstacles and preserving connectivity between initially connected robots.

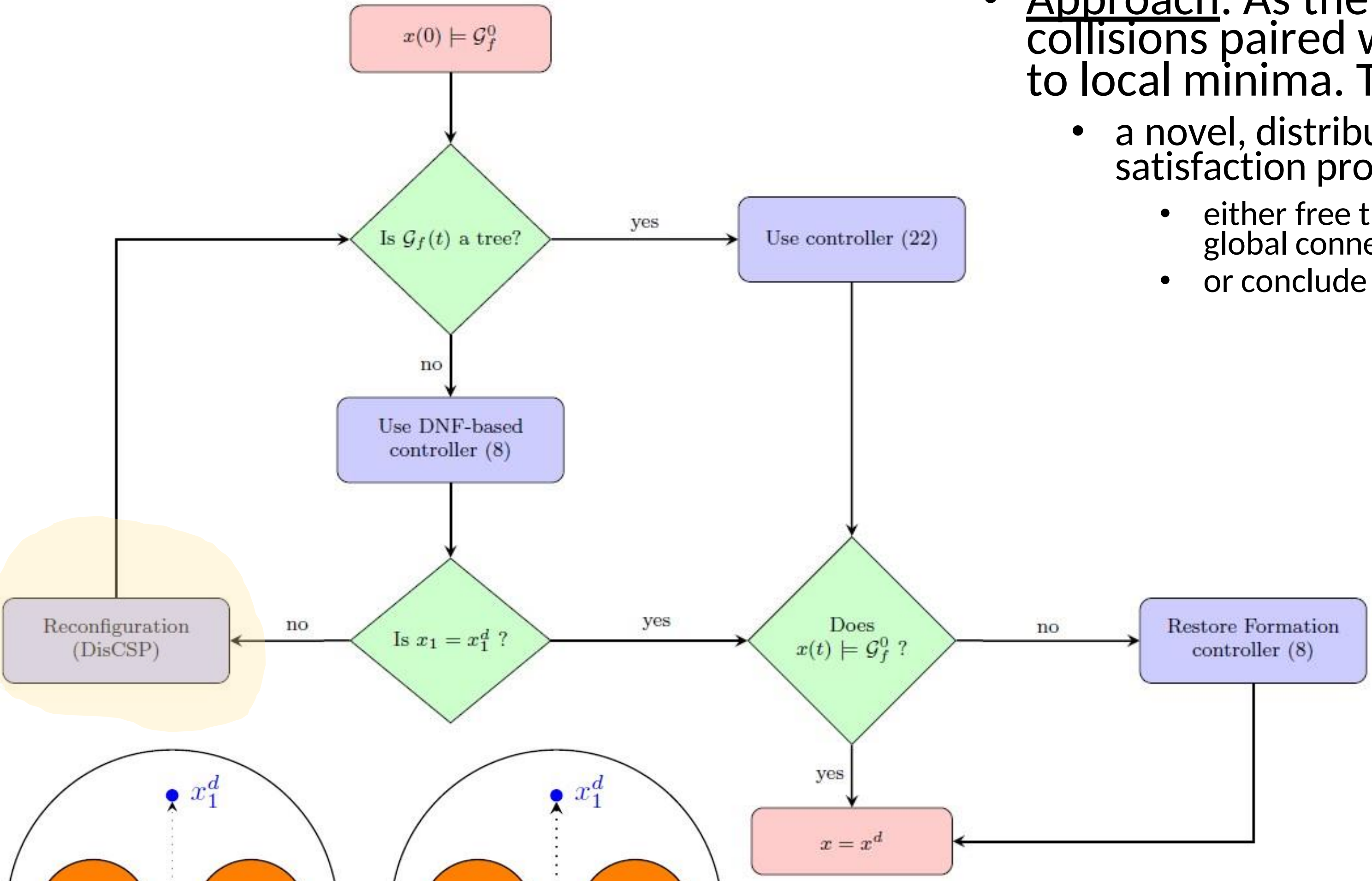


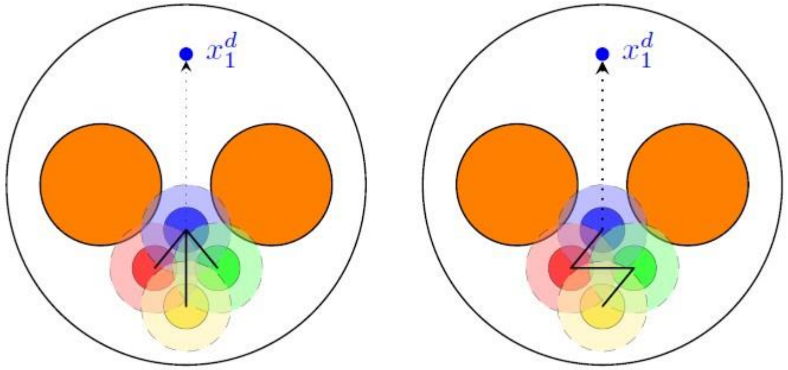
$$u_i(t) = -\nabla_i \varphi_i, \quad i \in \mathcal{V}$$

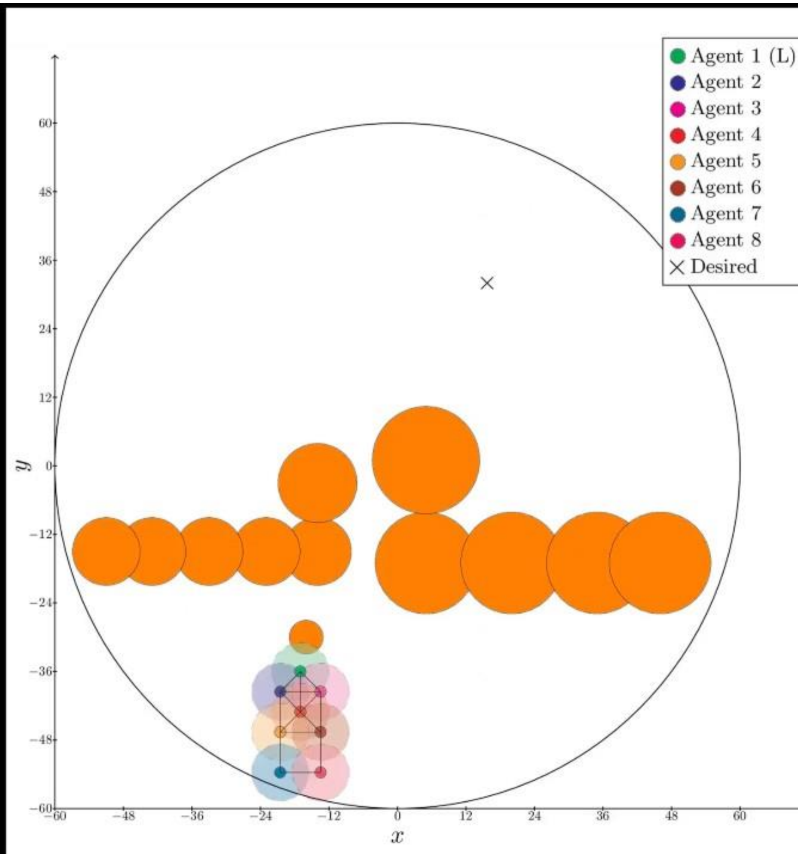


MultiAgent: Decentralized control of Holonomic Vehicles in Cluttered Environments

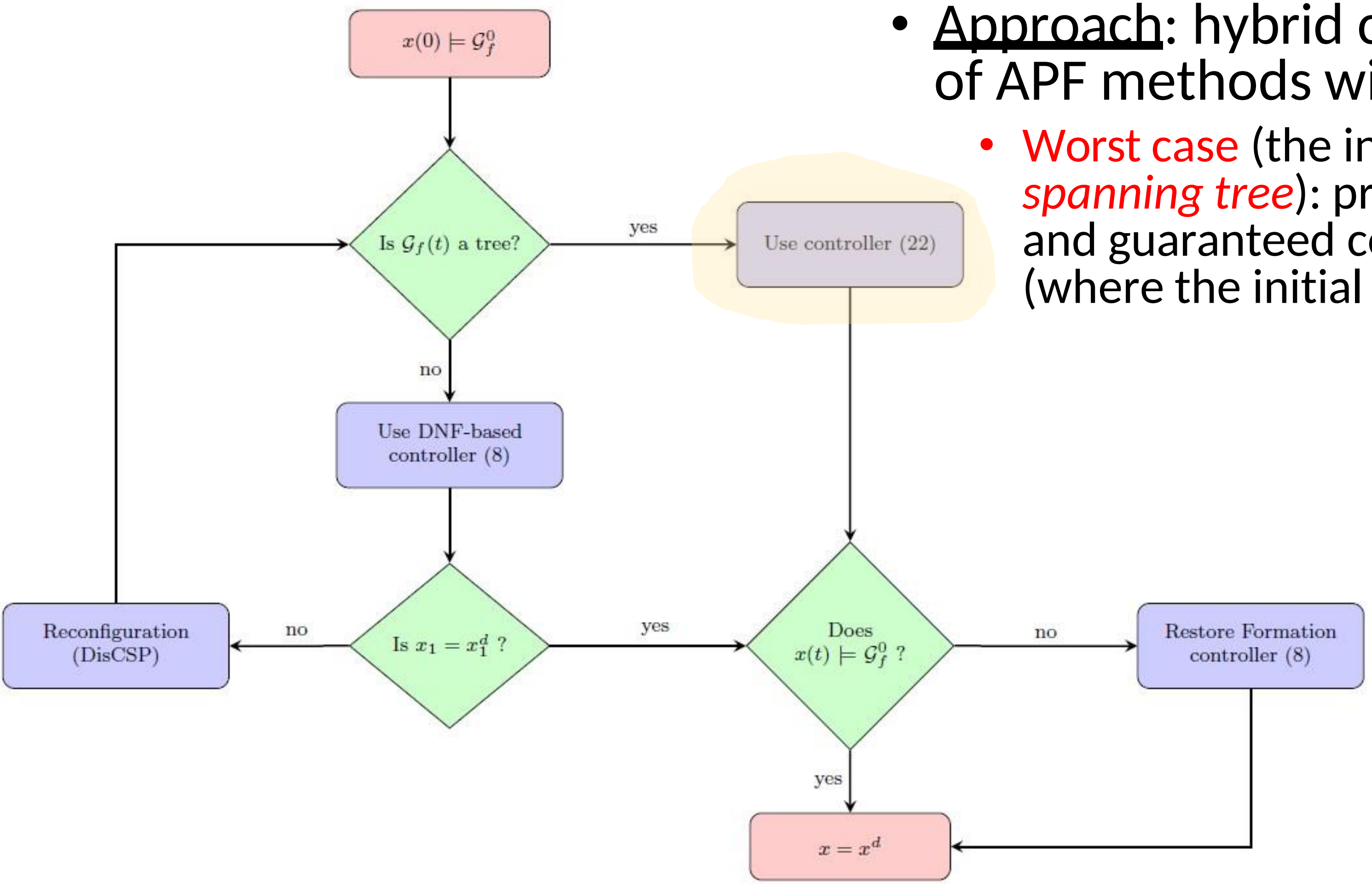
- Approach: As the system progresses to its goal, imminent collisions paired with connectivity violations may trap the system to local minima. To cope with this:
  - a novel, distributed, discrete, high-level algorithm (distributed constraint satisfaction problem on a local Voronoi partition), that is guaranteed to
    - either free the team through reconfiguration of the graph while maintaining global connectivity,
    - or conclude that the problem is infeasible as no further reconfiguration may occur.



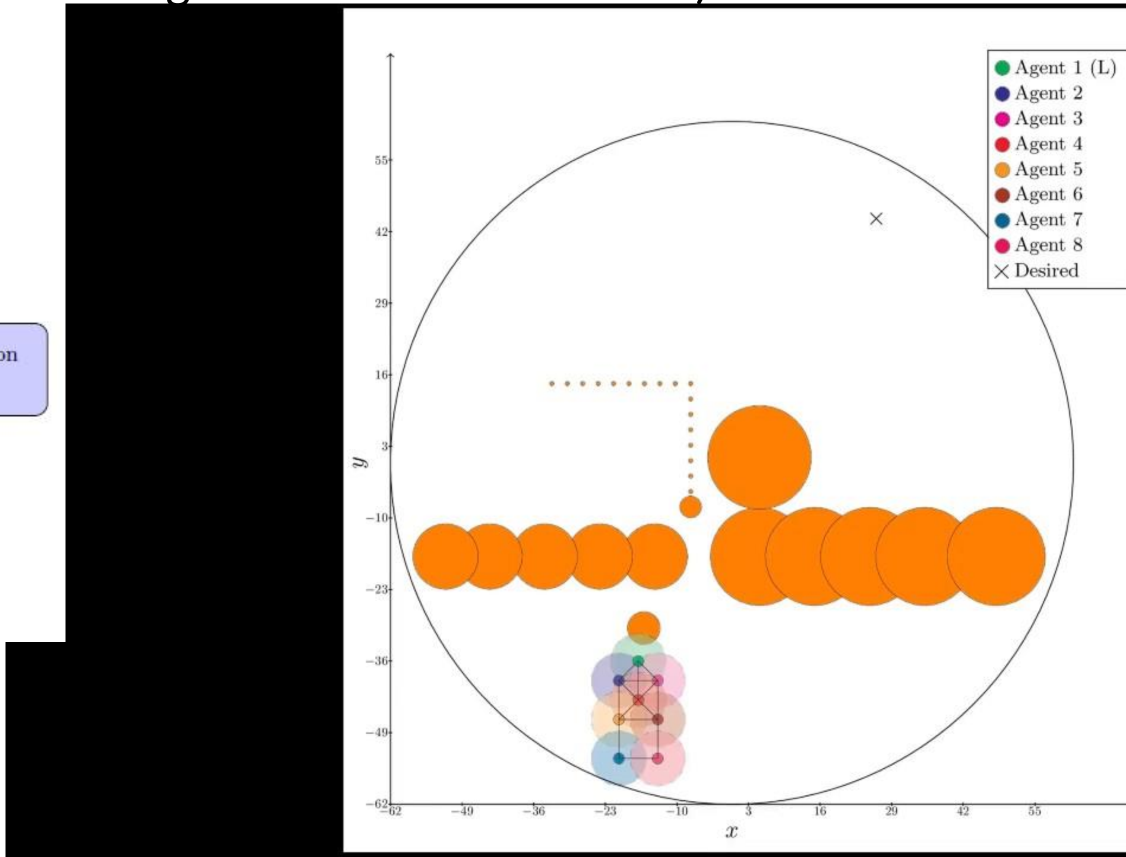




MultiAgent: Decentralized control of Holonomic Vehicles in Cluttered Environments

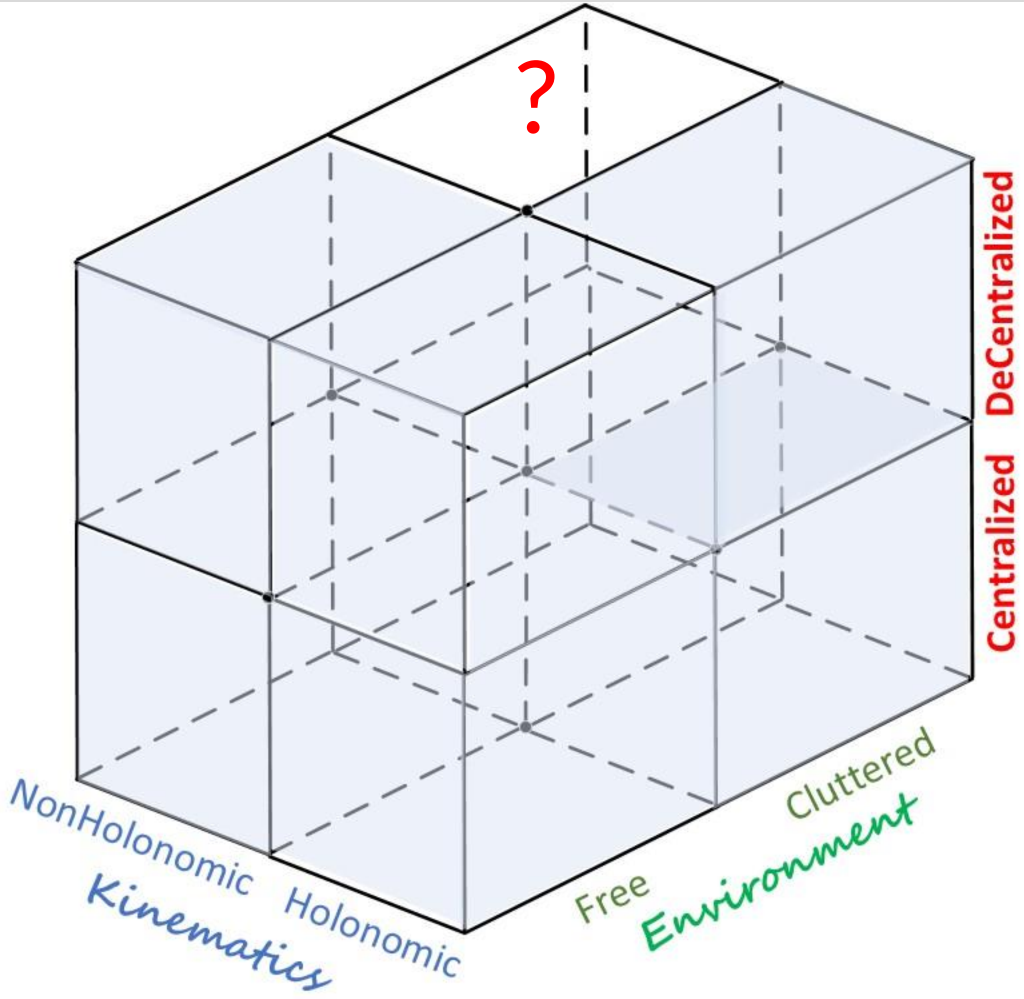


- Approach: hybrid control scheme, combining the simplicity of APF methods with a discrete reconfiguration algorithm:
  - Worst case (the initial graph will have been transformed into a *spanning tree*): prescribed performance control (PPC) with safe and guaranteed convergence to drive the team to its goal location (where the initial configuration will be restored)



- Reconfiguration in conjunction with the DNF controller may lead to the desired configuration, BUT no formal proof of convergence can be acquired.
- In search of a provably correct algorithm, we introduce a novel controller that can handle the case where the formation graph is a *tree graph* (worst case).

Motion Planning: The Multi - Agent case

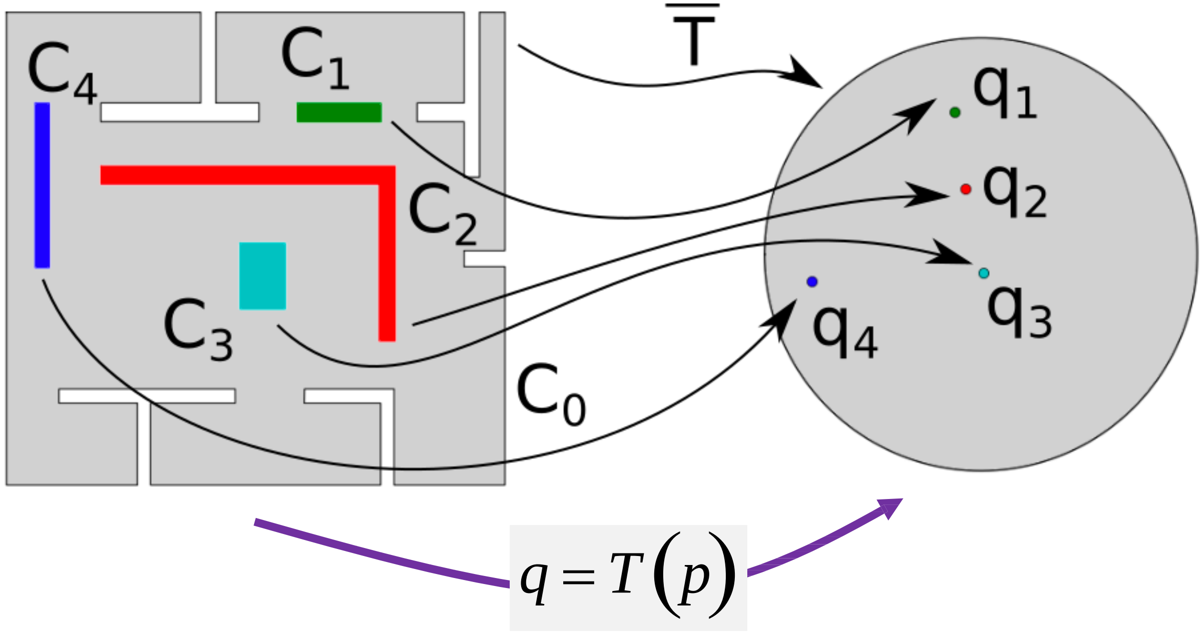


Motion Planning: The Single Agent case

Kinematics / Environment	HOLONOMIC	Non-HOLONOMIC
Free	D. Kodistchek & E. Rimon	
Cluttered		

Single Agent: Holonomic Motion Control in Cluttered Environments

- Problem: design a control law that can safely drive the robot to a specific goal configuration from almost all initial configurations.
- Features:
  - ➢ Workspace:
    - static,
    - compact,
    - planar, with arbitrary
      - ✓ connectedness and
      - ✓ shape.
    - A sufficiently fine polygonal workspace description needed (can be easily acquired in practice via SLAM).
  - ➢ Robot: disk shaped
- Approach:
  - ➢ Harmonic map (diffeomorphic transformation) of the workspace onto a punctured disk, i.e. a disk with some interior points removed (no need for decomposition of the workspace into trees of stars).



Panel Method (numerical technique) used to construct the proposed transformation $T(\cdot)$ (Harmonic Function).

- Simulations and experimentation to validate the efficacy of the proposed navigation strategy.

Single Agent: Holonomic Motion Control in Cluttered Environments

- Approach (contd.):
  - Adaptive Closed-form Artificial Harmonic Potential Fields (AHPF) for robot navigation to address:
    - ✓ the tradeoff between high obstacle repulsiveness, and
    - ✓ convergence rate to the desired position for almost all initial configurations .
  - A computationally expensive problem is solved only once for a given static workspace, independently of the robot's initial and goal configurations.

Adaptation to render: (a) the workspace outer boundary **repulsive** and b) any critical point of in the vicinity of the robot **non-degenerate**

Adaptation Law

$$\dot{k} = f\left(q, k\right)$$

$$\phi = k_d \ln\left(\frac{\|q - q_d\|}{2}\right) - \sum_{i=1}^{N} k_i \ln\left(\frac{\|q - q_i\|}{2}\right)$$

Desired Configuration

Inner Obstacles

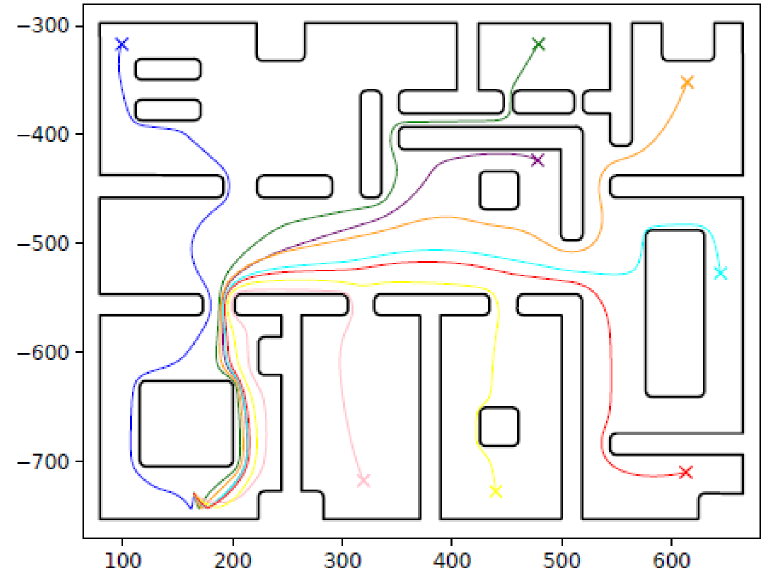
$$\psi = \frac{1 + \tanh(\phi / w)}{2}$$

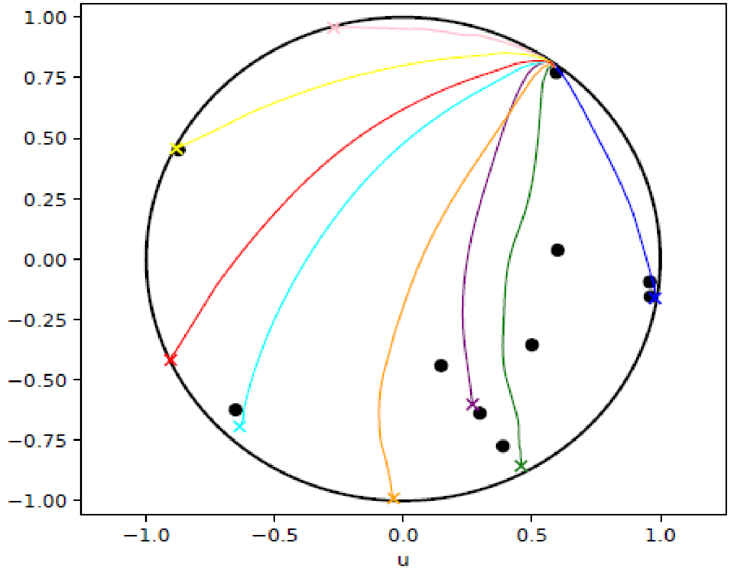
AHPF

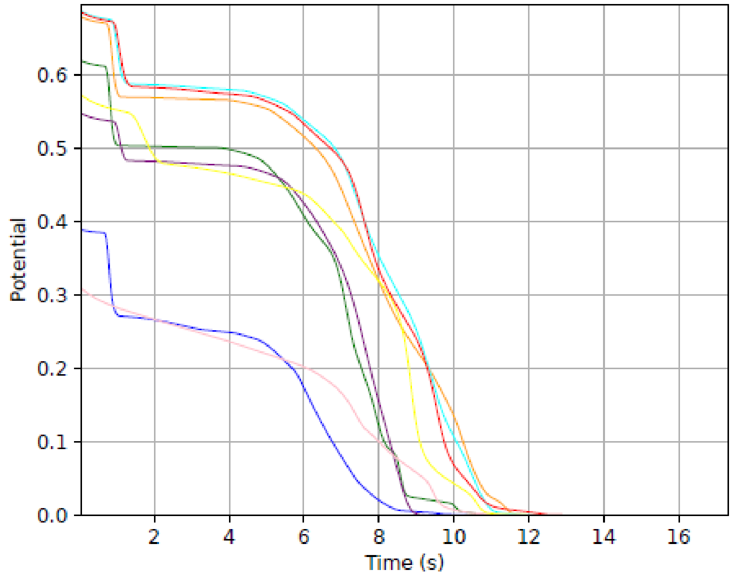
Control Law

$$u = -K_u s(q, k) J^{-1}(p) \nabla_q \psi(q, k)$$

Single Agent: Holonomic Motion Control in Cluttered Environments

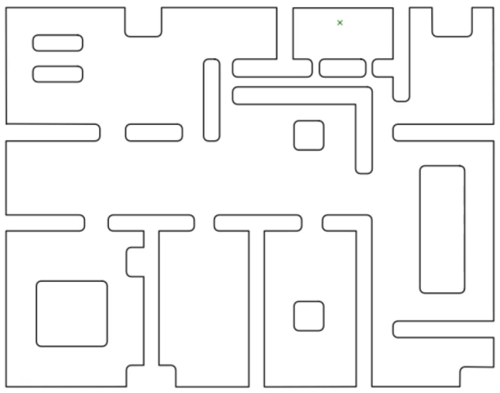


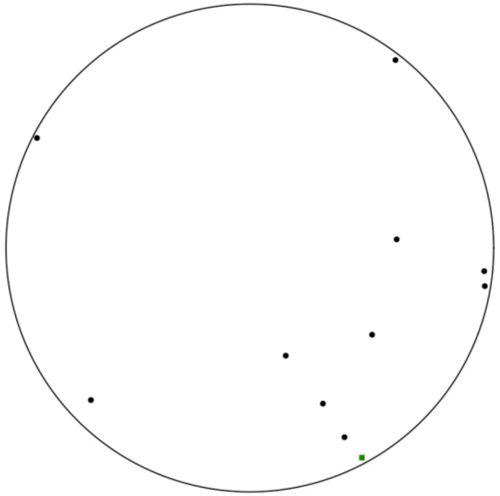




The evolution of ψ along the robot trajectories.

- Simulations to validate the efficacy of the proposed navigation strategy.
- **Full** (single) workspace transformation.
- The robot, each time, starts from an arbitrary location and navigates to different goal configurations (colored "×").

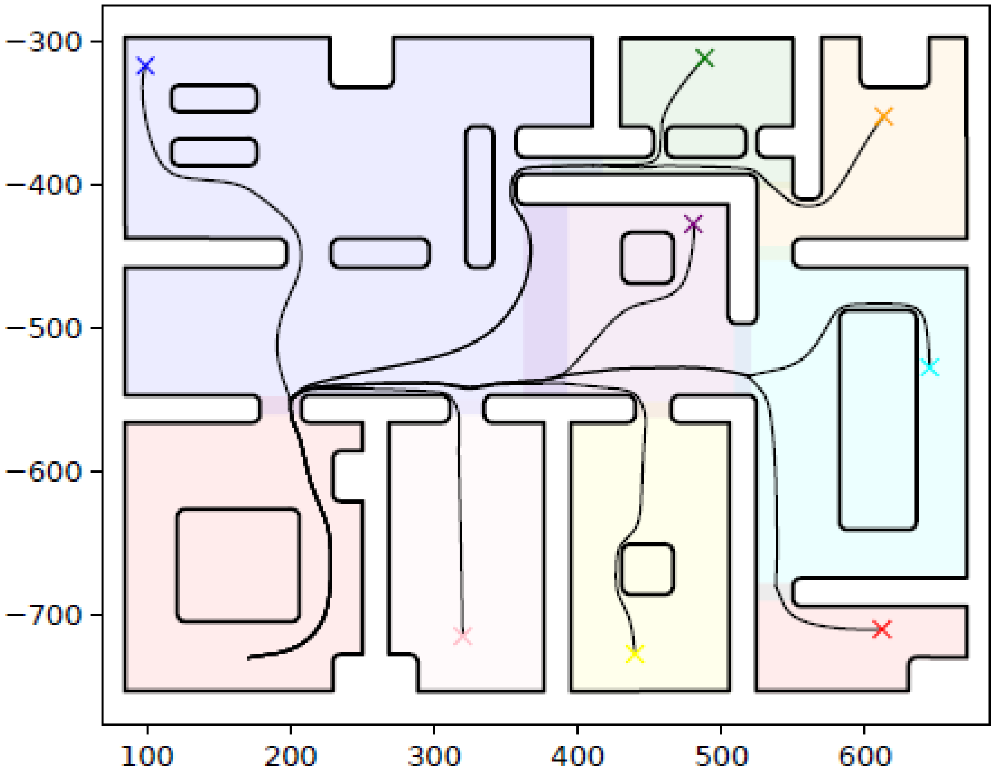


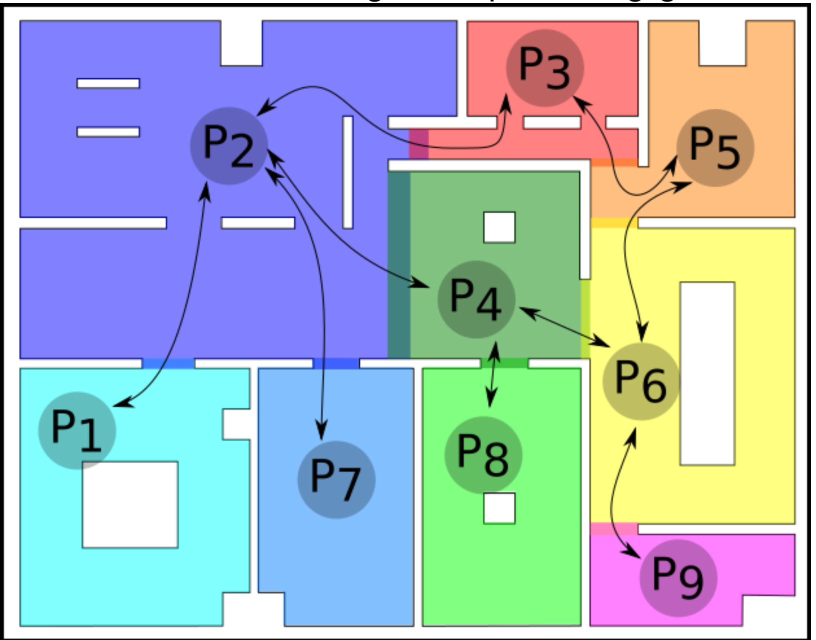


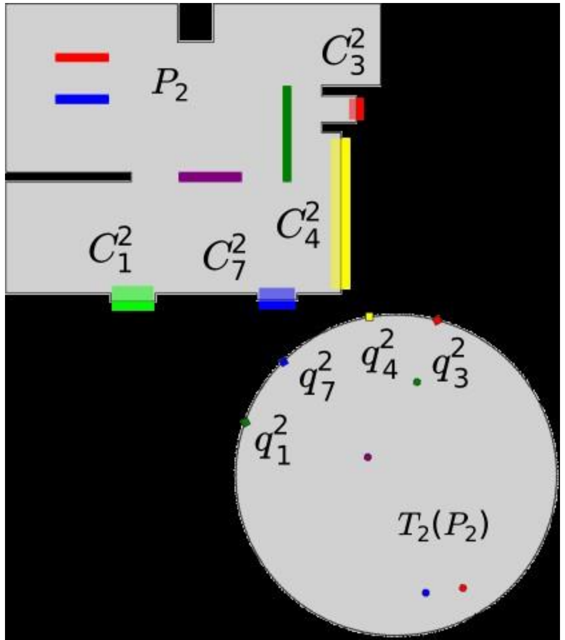
- SIMULATION SPECIFICS
- $T(p)$ and $J(p)$: C++
- Control & Adaption Laws: Python
- HW/OS : PC, Intel i5, 2.2Ghz, 4GB RAM, GNU/Linux.
- Simulation Method: Euler, 10 ms step
- Complexity Measures: for W described by 3680 segments
  - $T(p)$ and $J(p)$: 6 ms/step
  - Construction of $T(p)$: 5.4 s (once)

Single Agent: Holonomic Motion Control in Cluttered Environments

- As the size of the workspace W increases, the problem of computing the Harmonic Transformation T grows in complexity, since numerical techniques are polynomial in the number of elements used for representing W.
- To cope with large workspaces efficiently, we propose the construction of an atlas obtained by separating the W into N_A overlapping subsets P_i, and constructing a separate harmonic map T_i for each P_i.
- This allows us to solve many small (and computationally less intensive) problems instead of a large one, thus reducing the overall resources required for addressing W.
- Therefore, given such a partitioning of W, we define the graph $G = (V, E)$ with the set of corresponding nodes (workspace partitions) and the set of edges between the nodes, indicating a feasible transition from one partition to another.
- Thus, for a given atlas and initial and final configurations, we can employ standard graph search algorithms to obtain a sequence of indices corresponding to partitions that the robot can traverse to reach its goal.
- Note that, since the partitioning of W does not need to be fine, the size of G will generally be small and and the cost of finding the sequence negligible.

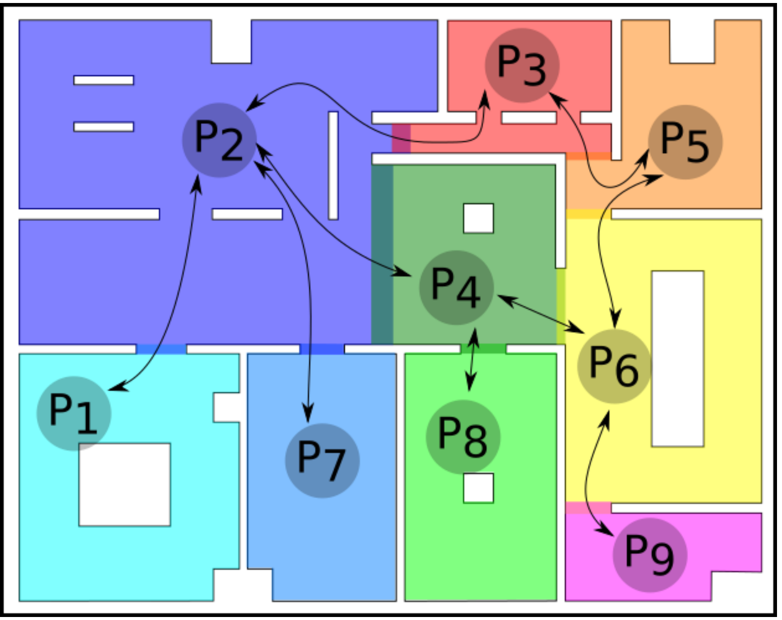


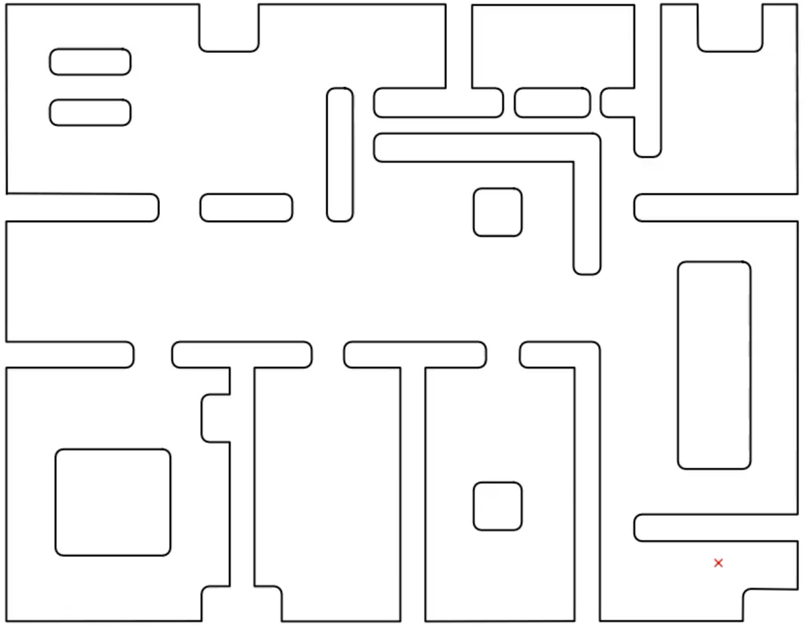


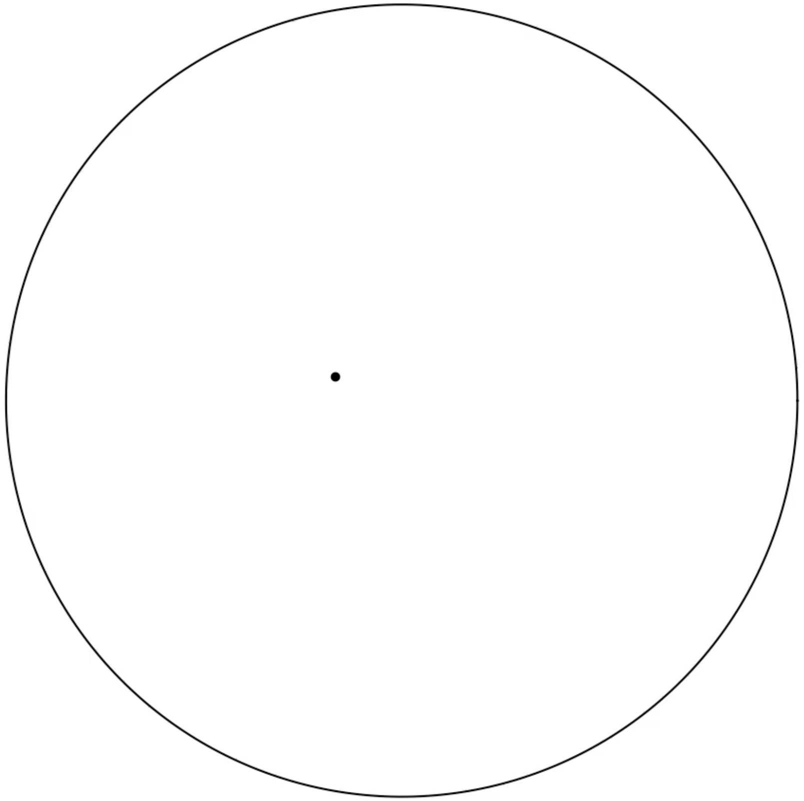


- SIMULATION SPECIFICS
- Complexity Measures: for P_i's described by 320 – 1000 segments (3680)
  - $T(p)$ and $J(p)$: 1.0 – 2.2 ms/step (6)
  - Construction of $T_i(p)$: 0.019 – 0.211 s (5.4)

Single Agent: Holonomic Motion Control in Cluttered Environments







Single Agent: Non-Holonomic Motion Control in Cluttered Environments

$$\dot{p} = \hat{n}_\theta v$$
$$\dot{\theta} = \omega$$

$$\hat{n}_{\tilde{\theta}} = \frac{J(p)\hat{n}_\theta}{\|J(p)\hat{n}_\theta\|}$$

$$\dot{q} = \hat{n}_{\tilde{\theta}} \tilde{v}$$
$$\dot{\tilde{\theta}} = \tilde{\omega}$$

$\hat{n}_{\tilde{\theta}} = \Upsilon \; \cos\theta \quad \sin\theta \; /_f^T$

Control Law in D

$$\tilde{v} = -\mathcal{K}_v s_v(q, \tilde{\theta}, k) \left(\hat{n}_{\tilde{\theta}}\right)^T \nabla_q \psi(q, k)$$
$$\tilde{\omega} = -\mathcal{K}_\omega \left(\hat{n}_{\tilde{\theta}}^\perp\right)^T \nabla_q \psi(q, k)$$

Control Law in W

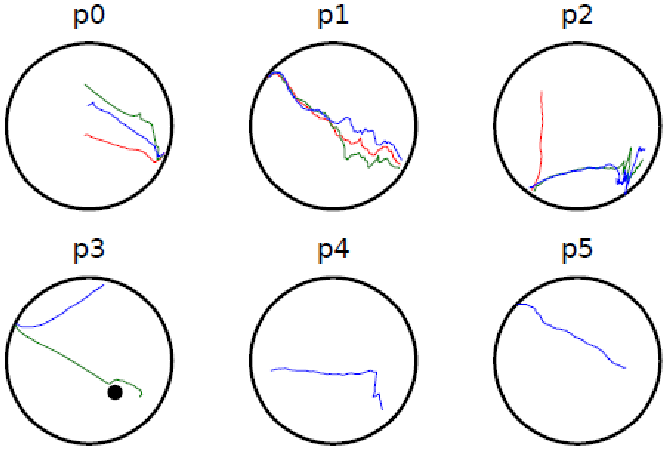
$$v = \nu \tilde{v}$$
$$\omega = \omega_{dq} + \omega_{d\tilde{\theta}}$$

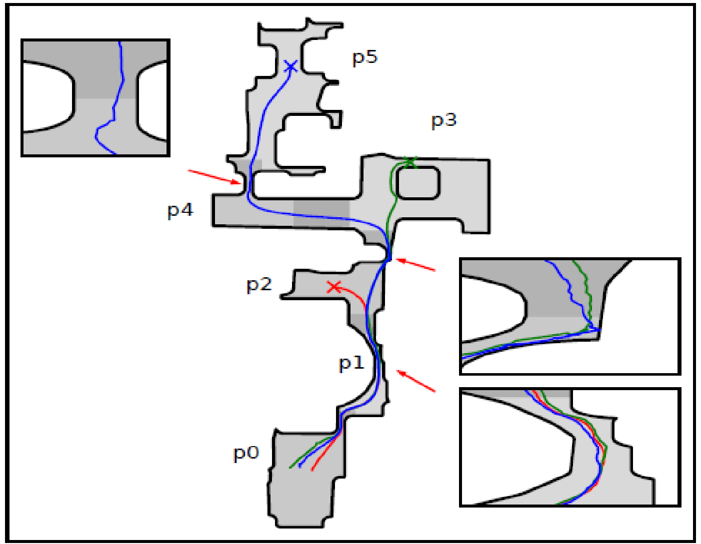
$$\omega_{dq} = -\tilde{v}\nu^2 \left(\left(J \begin{bmatrix} \hat{n}_\theta & \nu \hat{n}_{\tilde{\theta}}^\perp \end{bmatrix} \right)^{-1} \frac{\partial}{\partial \hat{n}_\theta} J \hat{n}_\theta \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\omega_{d\tilde{\theta}} = \tilde{\omega} \left(\left(J \begin{bmatrix} \hat{n}_\theta & \nu \hat{n}_{\tilde{\theta}}^\perp \end{bmatrix} \right)^{-1} \hat{n}_{\tilde{\theta}}^\perp \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\nu = \|J^{-1} \hat{n}_{\tilde{\theta}}\|$$

$\frac{\partial}{\partial \hat{n}_\theta} J$: directional derivative of J along $\hat{n}_\theta$





- Experiments:
- Platform: Robotnik SummitXL, nonholonomic
- Workspace:
  - boundaries obtained using readily available SLAM algorithms.
  - partitioned into six overlapping subsets
- Task: the robot was instructed to visit 3 different goal configurations (**position only**), each located in a different room.
- An off-the-shelf localization algorithm was employed to estimate the robot **position & orientation** using its on-board laser scanners and RBG-D cameras, providing feedback at approximately 5 Hz.
- Result: Our algorithm successfully managed to drive the robot safely to its specified goals
- Remark: Oscillating exhibited in the configuration space image (p1 and p2): attributed both to:
  - the relative slow update of the robot's pose estimation, and
  - the inversion of the Jacobian which is generally ill-conditioned close to narrow passages of the domain.

Experiments
Case 3

Motion Planning: The Single Agent case

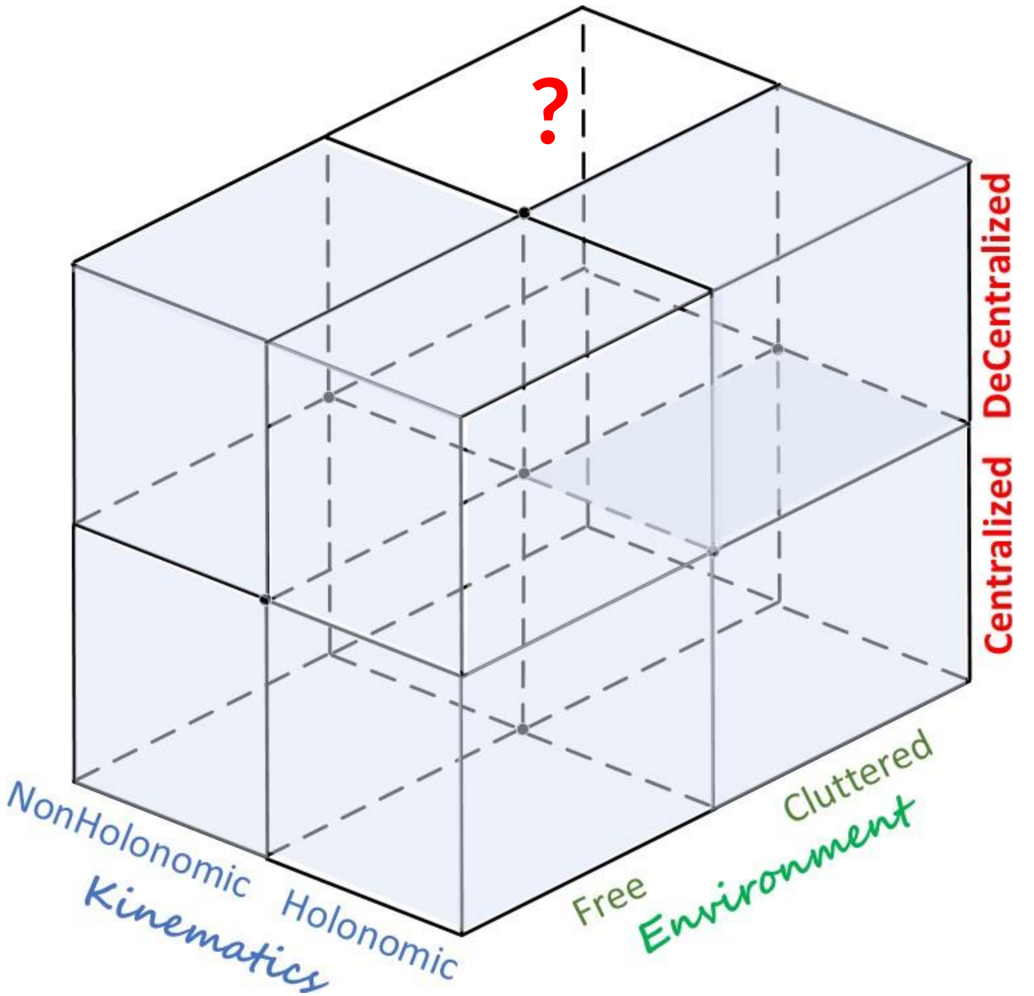
Kinematics / Environment	HOLONOMIC	Non-HOLONOMIC
Free	D. Kodistchek & E. Rimon	
Cluttered		

Provable Motion Planning: what have we achieved ?

Single Agent

Kinematics \ Environment	HOLONOMIC	Non-HOLONOMIC
Free	D. Kodistchek & E. Rimon	
Cluttered		?

Multi - Agent



Conclusion: what have we (not) achieved so far ?

- Achievement
  - A set of provable motion planning methodologies
  - Treatment of almost all combinations of:

$$Number\_of\_agents \times Environment\_type \times Kinematics \times Coordination\_Type =$$

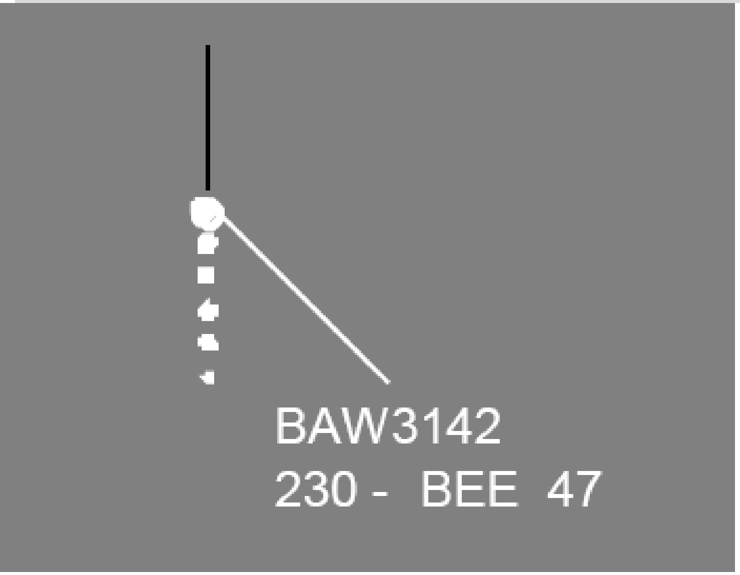
$$= \{\text{single, multiple}\} \times \{\text{free, cluttered}\} \times \{\text{holonomic, honholonomic}\} \times \{\text{centralized, decentralized}\}$$

- Pending:
  - Sensing radius: Constrained / Sector Bounded
  - Combination: MultiAgent - Cluttered – Nonholonomic – Decentralized?
  - Methodology: A generic methodology, uniformly treating all combinations, is still missing…

Extending and Applying our Provable Motion Planning Approaches

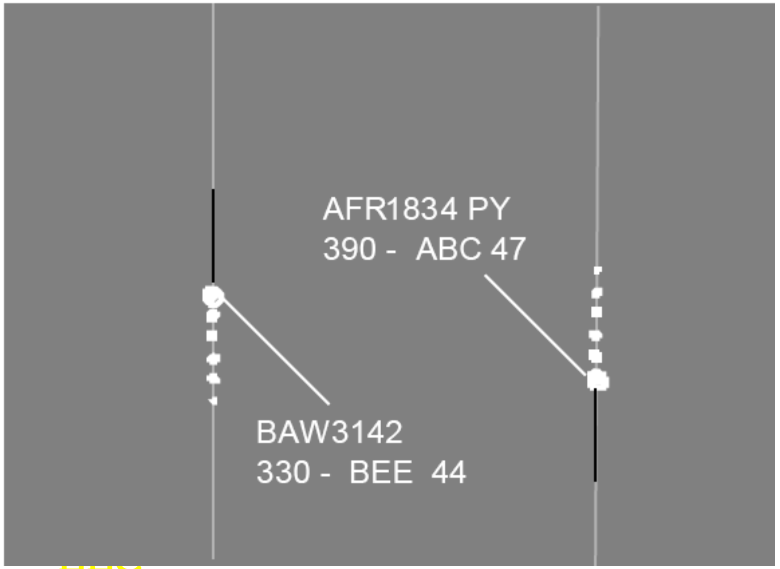
- Label Anti Overlapping
- Provable Exploration
- Multi-Agent Collaboration
- Prescribed Time Scale Robot Navigation in Dynamic Environments
- Collaborative Load Transportation
- Cooperative Underwater Manipulation
- Navigation Functions Learning

Label Anti-Overlapping Problem

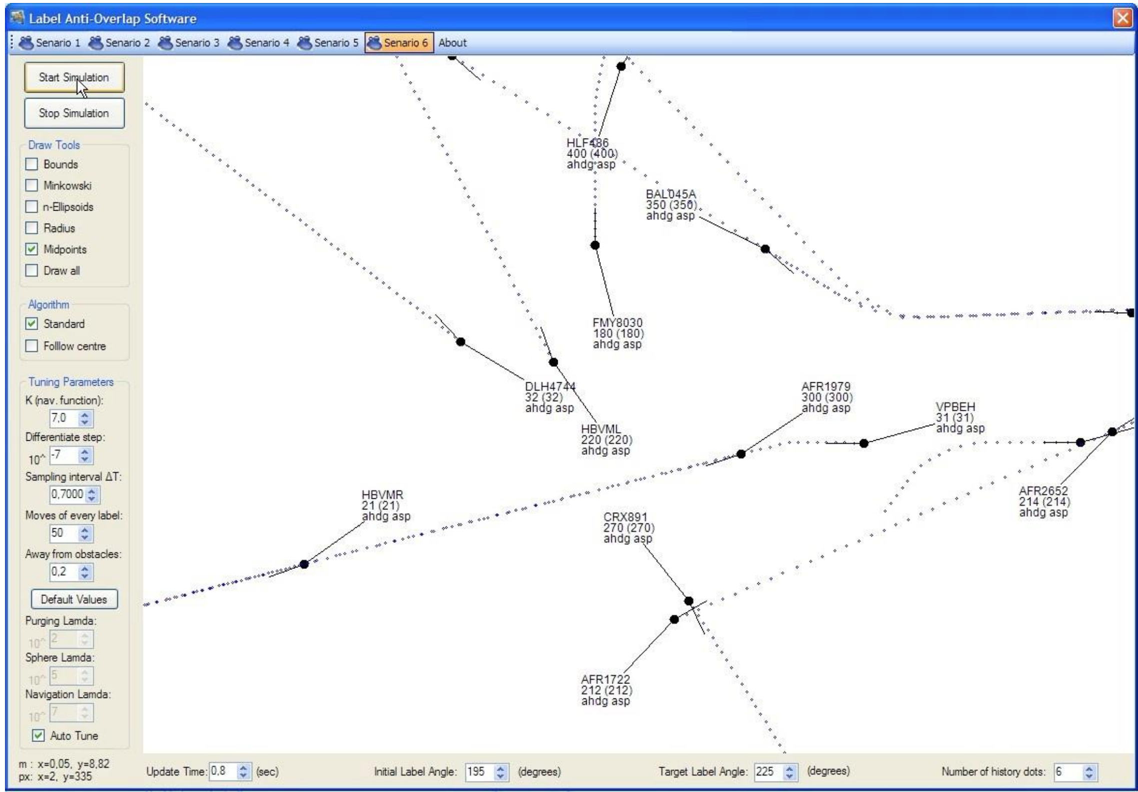


⇐ A typical "Label Structure" of an ATM Display

- The relationship between the aircraft position symbol and its label is established by means of a leader line connecting them.
- The 'selected' aircraft label being displayed as in-filled
- The connecting point between the leader line and the label could be any point located anywhere along an edge of the label, e.g. the midpoint or the corners of each edge.



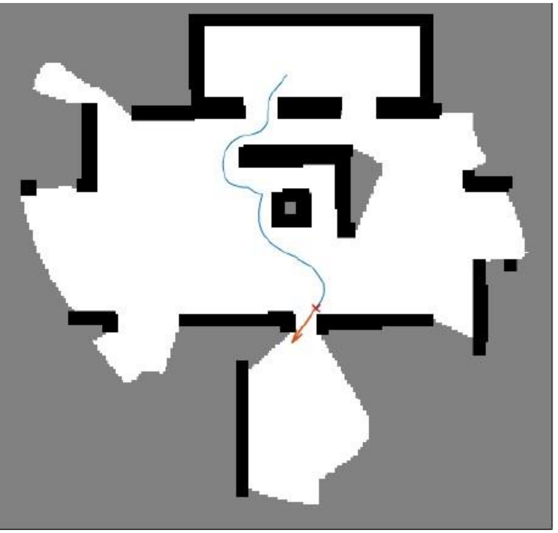
⇐ A typical "label collision course"



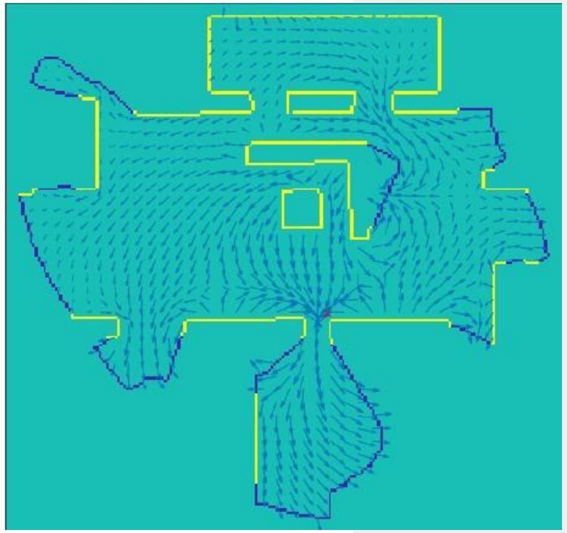
Provable Exploration

- Workspace $W \subseteq \mathbb{R}^2$: compact & connected
- Robot at $p \in W$: $\dot{p} = u$
- Robot equipped with proximity sensors allowing it to sense a disk, of non-occluded points, around it.
- A point of the workspace is considered as explored if, at any time, it belongs to the robot's sensing region.
- **Problem**: Find u that guarantees exploration of W in finite time.

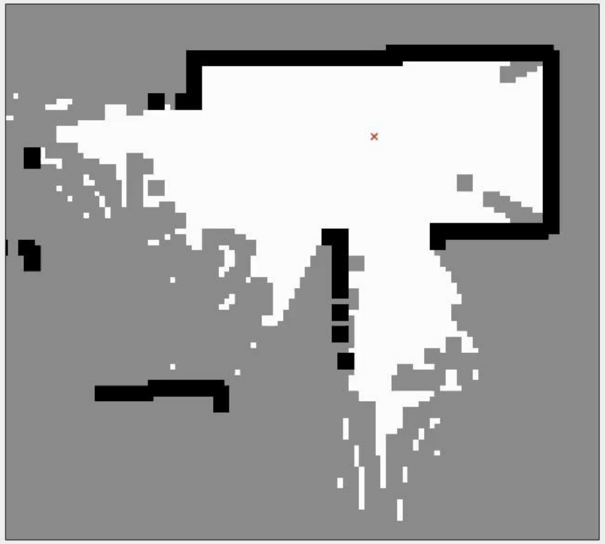
- At each time t:
  - Consider the boundary $\partial\mathcal{E}$ of the explored region.
  - Build & **update** a harmonic potential field ϕ with appropriate boundary conditions, such that:
    - The free part of $\partial\mathcal{E}$ is attractive
    - The occupied part of $\partial\mathcal{E}$ is repulsive
- Laplace equation solved via Fast Multipole-accelerated Boundary Element Method ⇒ computational cost: $O(n)$, instead of the usual $O(n^2)$. (n: number of boundary elements)
- We derived adaptive laws to ensure: (i) absence of undesired stable equilibria, (ii) convergence and (iii) map quality.

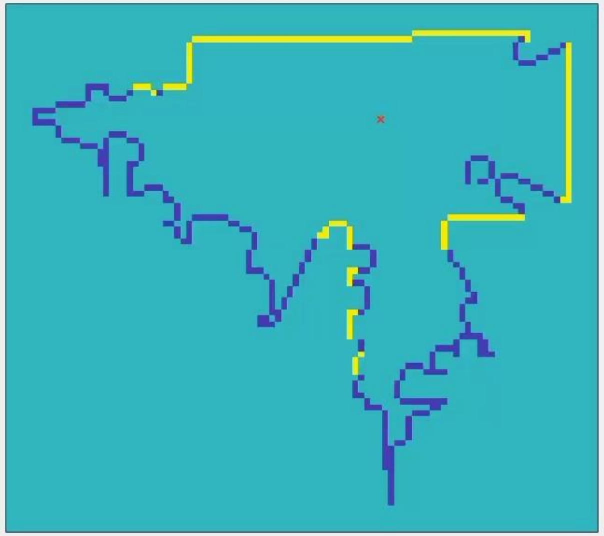


Occupancy grid map of the explored region.



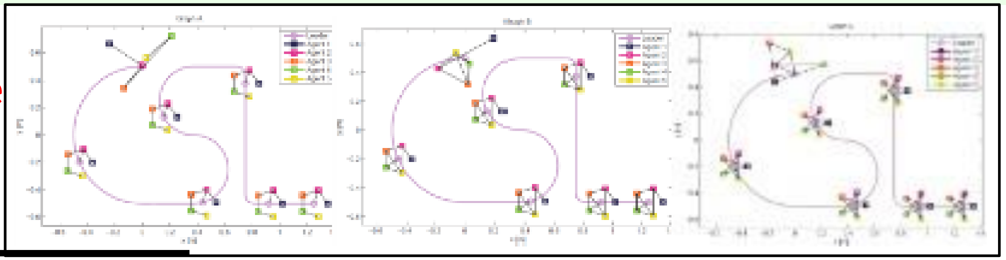
Boundary of the explored region and the induced vector field.



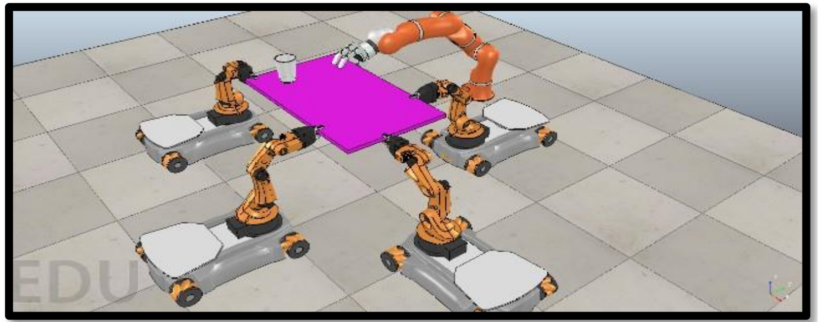


Multi-Agent Collaboration: Soft & Hard Constraints

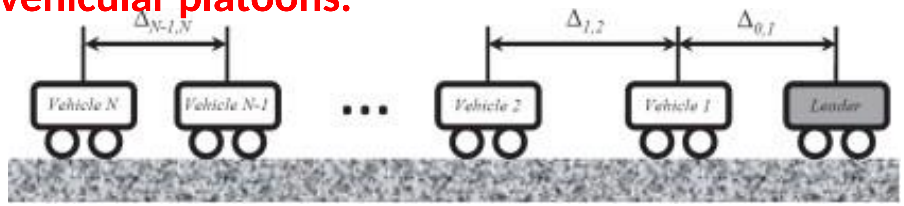
- **Formation control - prescribed performance**



- **Collaborative transportation via implicit communication.**



Robust control of large vehicular platoons.

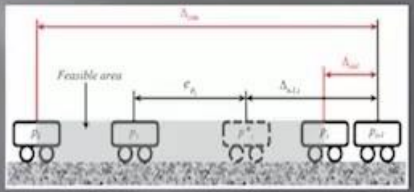


Summary:

1. Platoon:
   - Leader: Pioneer2AT
   - Followers: 2 KUKA Youbots, 2 Pioneer2DX
2. Feedback obtained by mounted infrared sensors.

Control Objectives:

1. Achieve a desired inter-vehicular distance 0.2m.
2. Avoid collision between successive vehicles.
3. Avoid connectivity breaks due to sensor limitations.
4. Achieve prescribed transient and steady state response.



Summary:

1. Two Pioneer mobile robots in a Leader-Follower scheme.
2. Compliant contact between the object and the follower.

Control Objective:

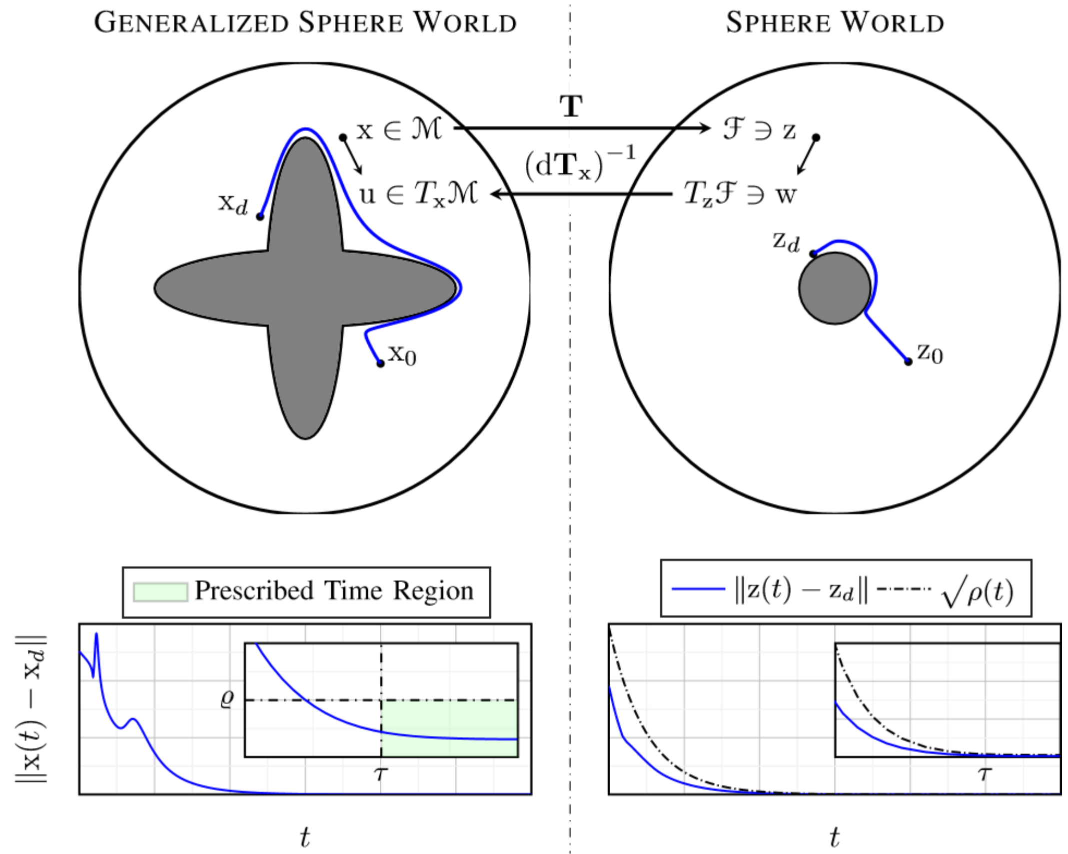
Achieve a desired configuration known only to the leader.

Properties:

1. No explicit information is communicated.
2. The follower measures the exerted force and torque at the contact.
3. The closed loop is affected by certain designer specified performance functions.

Prescribed Time Scale Robot Navigation in Dynamic Environments

Single Robot Navigation in the presence of moving obstacles with predetermined arrival time at a neighborhood of the desired configuration



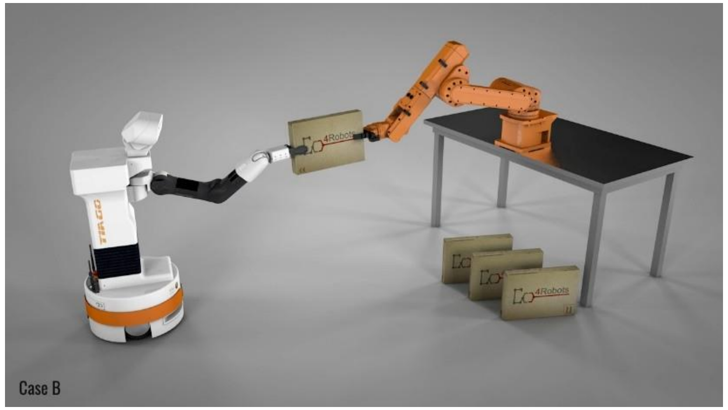
Features

- ✓ Closed-loop velocity controller based on the Prescribed Performance Control Methodology for sphere worlds.
- ✓ Guaranteed convergence and collision avoidance.
- ✓ Extension to a wide class of workspaces through diffeomorphic transformations.
- ✓ Changes in the topology of the configuration space can be accounted under very mild assumptions.

Collaborative Load Transportation

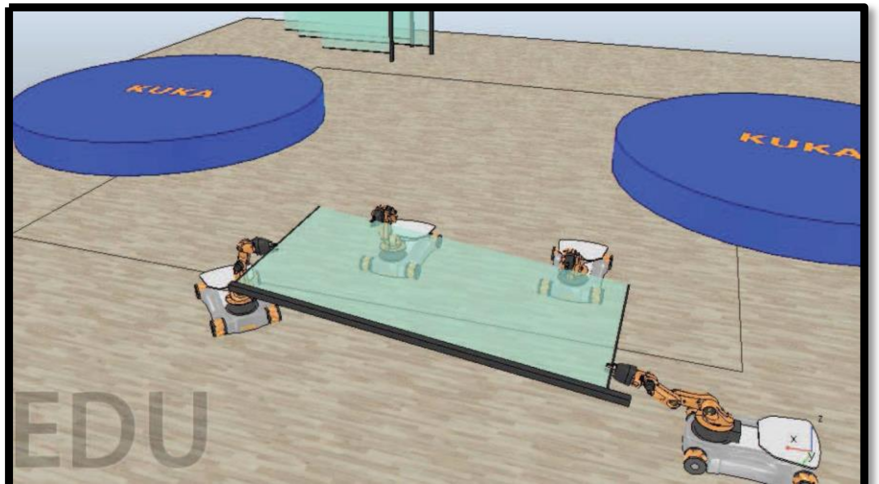
Human - Robot

Robot - Robot



Features:

- ✓ A **leader-follower** scheme for the cooperative transportation of the object is implemented.
- ✓ The **human** (or a **robot**) acts as the leader and the **mobile manipulator** obtains the role of the follower.
- ✓ The human (or a robot) leads the way and the robot (follower) perceives the **motion intention** of the leader implicitly, using the measurements from a F/T sensor mounted on its wrist.
- ✓ A proper **force feedback control scheme** is implemented for the motion of the mobile manipulator.
- ✓ The mobile manipulator is **compliant** with the human achieving a safely object transportation.
- ✓ Both the Leader and the Follower are endowed with **collision avoidance** capabilities.



Collaborative Load Transportation

Scenario

The scenario revolves around three (3) robotic entities - a mobile manipulator, a static manipulator and a mobile robot – interacting with each other as well as with an object that can be grasped, and the environment.

Heterogeneous Agents:

- Static Manipulator (Mitsubishi PA 10-7C)
- Mobile Manipulator (PAL Robotics TIAGo)
- Mobile Platform (Robotnik SUMMIT-XL HL)
- Humans

Workspace Description:

- Loading Region (R1)
- Unloading Region (R2)
- Surveillance workspace 1 (R3)
- Surveillance workspace 2 (R4)

Objects:

- Heavy Box (HB)
- Light Box (LB)

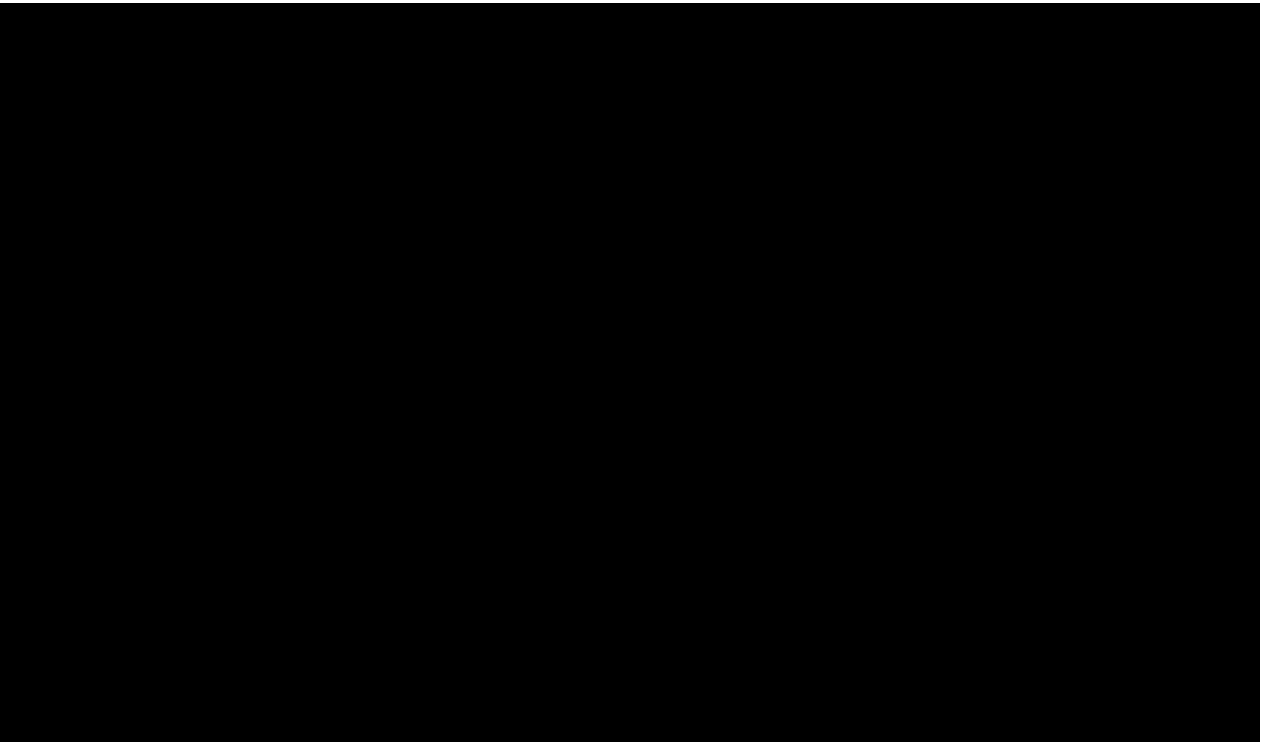
Cooperative Underwater Manipulation

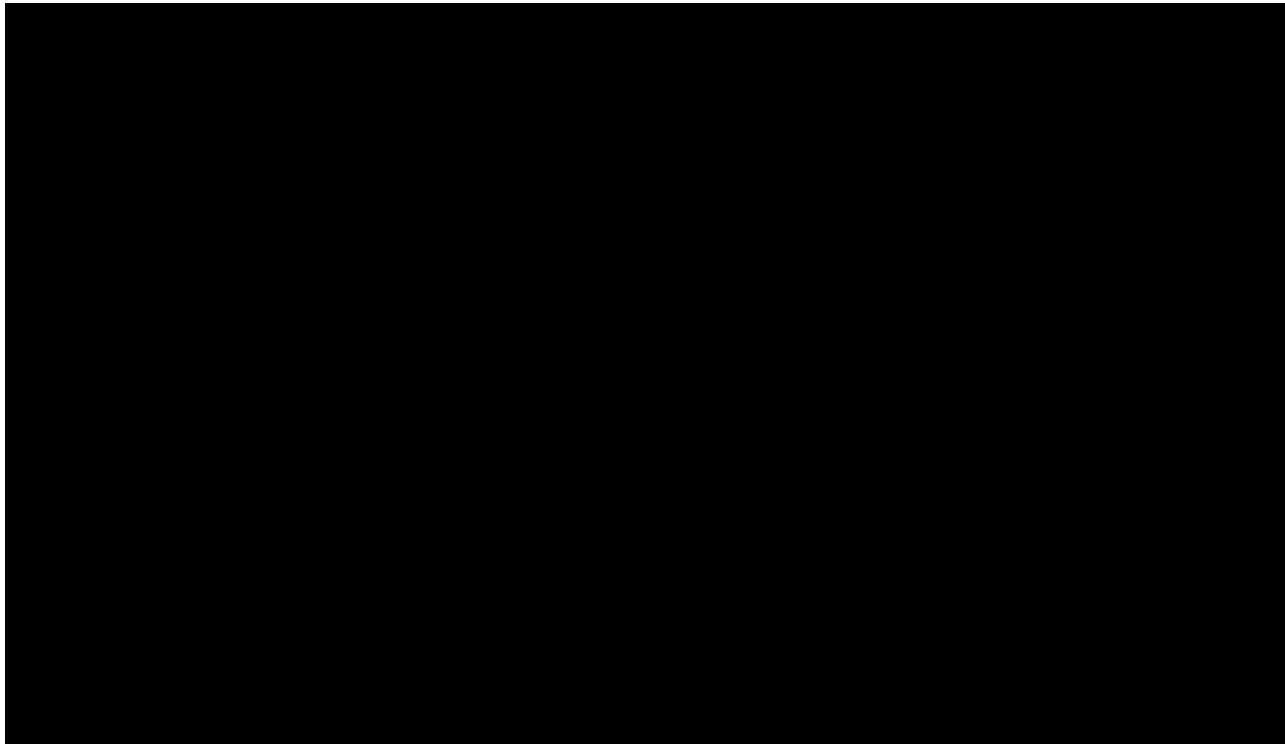
Approach

- A cooperative object transportation scheme for Underwater Vehicle Manipulator Systems
- Safe motion of the team towards the goal configuration.
- Implicit communication, thus avoiding completely tedious explicit data transmission.
- Only the leading UVMS is aware of
  - the desired configuration of the object and
  - the obstacles' position in the workspace
- The followers
  - estimate the object's desired trajectory and
  - implement an impedance control law
- The proposed scheme adopts load sharing among the UVMSs according to their specific payload capabilities.

Future Efforts

- Experimental Verification (performed as we speak…)
- UVMSs with Underactuated Vehicle Dynamics.





Navigation Functions Learning from Experiments: Application to Anthropomorphic Grasping

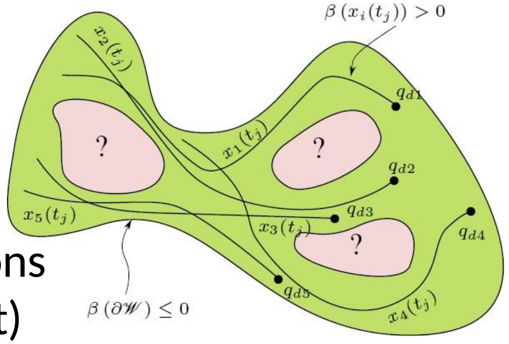
Usual, "Direct" Approach: Given some obstacles, navigate in collision-free manner

Inverse Motion Planning: Infer "Obstacles" from Observed (collision-free) Trajectories

$$\varphi ! \frac{\gamma_d}{\sqrt[k]{\gamma_d^k + \beta}}$$

Motivation 1:
Inverse Motion Planning

- Learn unknown obstacles
- Generalize to other destinations
- Feedback Motion Plan (robust)



Motivation 2:
Anthropomorphism

- Learn human movement
- Nonlinear Learning Capability
- Continuous smooth trajectories (natural)

Problem Statement:

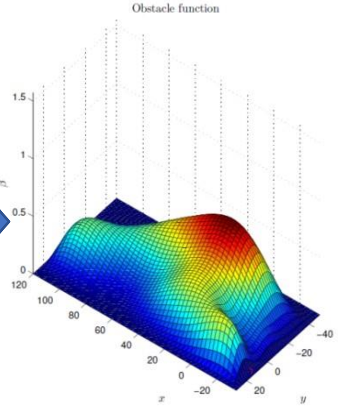
- Assume that
  - *Positions $x_i(t_j)$* ,
  - *Velocities $u_i(t_j)$* and
  - *Destinations q_{di}*

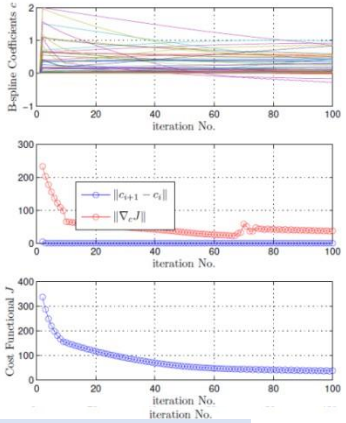
  are available after experiments.
- Find the obstacle function $\beta(q)$ satisfying:

$$u_i(t_j) = -\nabla_q \varphi(x_i(t_j), q_{di})$$

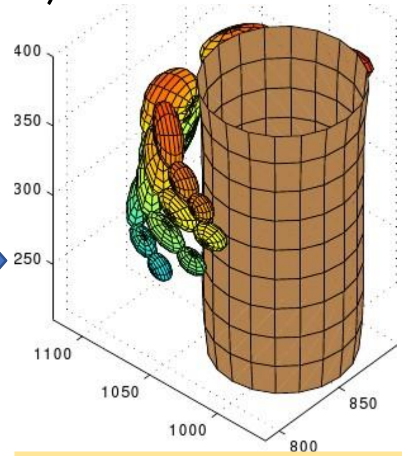
- Solution: Leads to PDE, solved via optimization over B-Spline Coefficient Space

Grasping Experiments





Learning Algorithm



Navigation Function

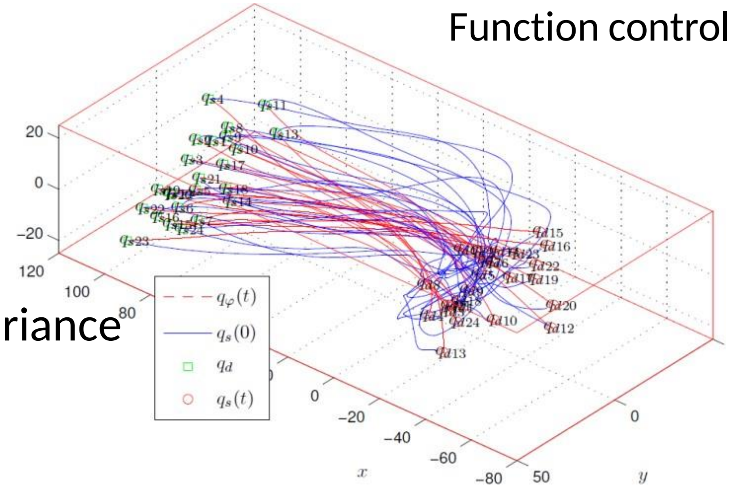
Question: *Can we construct fictitious obstacles leading to the observed trajectories?*

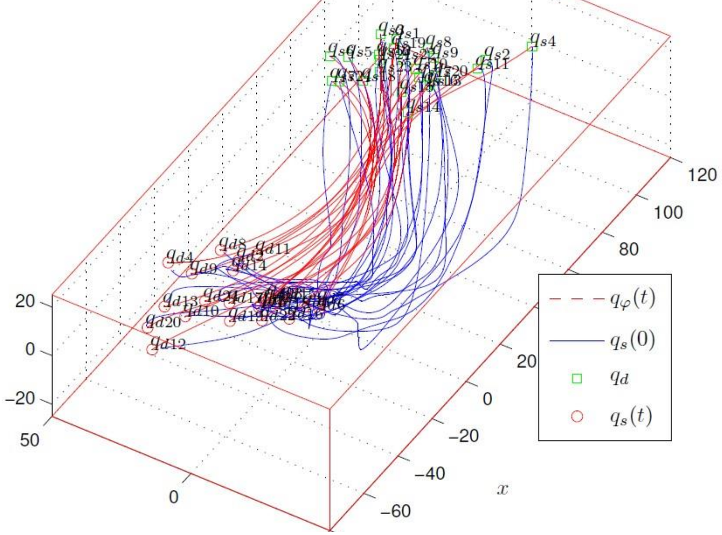
Navigation Functions Learning from Experiments: Application to Anthropomorphic Grasping

- Experiments provided the data
- PDE solution provided an obstacle function
- Destinations are selected as in the original experiments

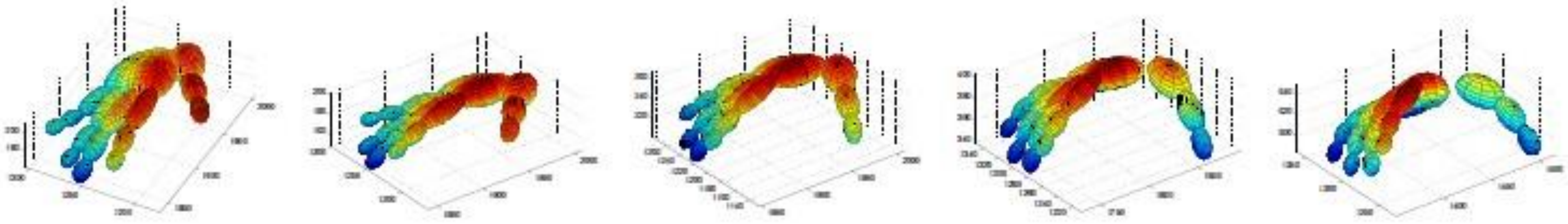
1. Principal Component Analysis
2. Select number of Principal Components based on variance
3. Train the Navigation Function in *this* subspace of the configuration space
4. Control the system in this subspace

Comparison between experiments and Navigation Function controller simulation

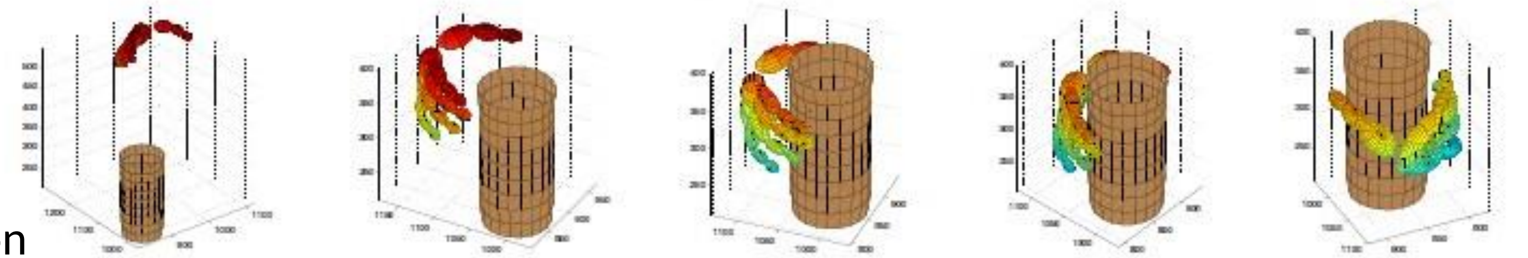




Automatically generated grasping movement using the Navigation Function.
The hand is controlled in the 3-dimensional configuration subspace.



- Grasping a tall glass
- Anthropomorphism reproduced using a small number of principal components
- The learnt “Virtual” obstacle results in this motion



Questions ?